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Examination for the Master's Course Methods of Econometrics winter semester 2022/23

Problem 1 (35 points)

A physics-based model for the energy consumption y of electrical vehicles (EV) per kilometer [kWh/km] at constant speed v [km/h] on level roads is given by

$$y = \beta_0 + \beta_1 \frac{1}{v} + \beta_2 v^2 + \epsilon, \quad \epsilon \sim \text{ i.i.d. } N(0, \sigma^2).$$

An ordinary least-squares calibration on ten data points gives the estimation (the notation $\mathrm{e}-5$ denotes $*10^{-5}$ and so forth)

$$\hat{\beta}_0 = 0.08, \quad \hat{\beta}_1 = 2.0, \quad \hat{\beta}_2 = 1.1\mathrm{e}-5$$

and the variance-covariance matrix

$$\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}}) = \begin{pmatrix} 0.0011 & -0.02 & -1.5\mathrm{e}-8 \\ -0.02 & 1.0 & -1.2\mathrm{e}-7 \\ -1.5\mathrm{e}-8 & -1.2\mathrm{e}-7 & 1\mathrm{e}-11 \end{pmatrix}$$

(a) Identify the three factors x_1, x_2, and x_3.

(b) Test the null hypotheses (1) $H_{01} : \beta_1 = 0$ and (2) $H_{02} : \beta_1 \geq 1$.

(c) Calculate the expected energy consumption per kilometer at 30 km/h, 50 km/h, 100 km/h and 130 km/h.

(d) Give a general expression for the estimated variance of the energy consumption estimate as a function of the speed and evaluate this expression for a speed of 50 km/h.

(e) At which speed this EV consumes the least energy per kilometer? Argue that this optimal speed must be always greater than zero.

(f) A comparison of the three factors with physical facts reveals that β_1 is directly the average basis power demand in kW for heating, ventilation, radio, lights and so on. Larger cars have a basic power demand of about 3 kW and small EVs of 1 kW while the other parameters change little. Calculate the optimal speed for minimum consumption for large (small) EVs.

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Problem 2 (30 points)

Consider the CO_2 footprint of a normal passenger bus. We have following material items for making the bus and repairs:

- Iron/steel: 12 000 kg (CO_2-neutral recycling: 25 %)
- Aluminum: 500 kg (recycling 50 %)
- Plastic: 2 500 kg (recycling 20 %)
- Rubber: 500 kg (recycling 20 %)
- Other materials: 1 000 kg (recycling 20 %)

During operation, the bus consumes on average 50 l Diesel per 100 km. It has an average lifetime of 1.2 million km and carries, on average, 12 passengers (including trips without passengers). The fuel has a W2T CO_2 emission factor of 0.4 kg/l and a T2W factor of 2.7 kg/l

(a) Calculate the combined material and fuel repository y^{s} for all three life phases.

(b) The line of the emission factor matrix of the five materials (in above order) for the pollutant $e_1 = CO_2$ reads

$$\boldsymbol{C'} = (2.5, 25, 1, 1.5, 2)$$

Calculate the total CO_2 emissions e_1^{mat} for making and recycling the bus (without driving it).

(c) Calculate the CO_2 emissions per kilometer of driving.

(d) After which number of kilometers the driving emissions are the same as the total emissions in the production and recycling phases?

(e) Estimate the CO_2 footprint per passenger-kilometer

  (i) considering the driving phase, only,

  (ii) considering all three life phases.

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Problem 3 (55 points)

In order to assess the factors when deciding between buying a battery-electric vehicle (BEV) or an internal-combustion engine (gasoline or Diesel) vehicle (ICEV), each person of a group of ten persons should decide whether to buy a BEV (alternative $i = 1$) or an equivalent ICEV ($i = 2$). The only differences lie is the buying costs C_i (including possible subsidies) in multiples of 1 000 €, the kilometer costs C'_i in €/100 km due to fuel and electricity prices, and the range R [km] and recharging time [minutes] of the BEV. The survey resulted in following decision numbers y_1 and y_2 ($y_1 + y_2 = 10$):

Situation	C_1	C_2	C'_1	C'_2	R	T	y_1	y_2
1	30	30	20	20	300	180	4	6
2	30	30	20	20	300	360	0	10
3	50	30	20	20	300	180	1	9
4	60	30	20	20	300	180	0	10
5	40	30	10	20	400	180	7	3
6	30	30	10	20	500	120	9	1
7	30	30	20	10	200	360	1	9
8	20	20	20	20	100	180	2	8

(a) Does this survey follow a revealed or a stated preference design? Justify your answer.

(b) In order to be accessible for a discrete-choice analysis, the choice set must be complete and unique. Explain, what this means in this specific situation of buying cars.

(c) Why is it not sensible, in this specific situation, to also include the range and the refueling time of the ICEV into the choice set?

(d) The decisions are now analyzed by a binary Logit model with following utility function:

$$V_i = \beta_1 \delta_{i1} + \beta_2 C_i + \beta_3 C'_i + \beta_4 R_1 \delta_{i1} + \beta_5 T_1 \delta_{i1}$$

where $\delta_{ij} = 1$ if $i = j$ and =0, otherwise. Classify the five factors into alternative-specific constants, characteristics of the alternatives, and socioeconomic variables and give their meaning.

(e) Calculate the realized property sums

$$X_m^{\text{data}} = \sum_n \sum_i x_{mni} y_{ni}$$

for the parameters β_1 ($m = 1$) and β_5, and the corresponding expected property sums

$$X_m^{\text{MNL}}(\boldsymbol{\beta}) = 10 \sum_n \sum_i x_{mni} P_{ni}$$

for the trivial model with $\hat{\boldsymbol{\beta}} = \mathbf{0}$. (Do not calculate the other property sums!) Why $\hat{\boldsymbol{\beta}} = \mathbf{0}$ is not a ML estimator for the given data?

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(f) The ML estimates parameters are now given as (value ± standard deviation $\sqrt{V_{jj}}$)

$$\begin{aligned}
\hat{\beta}_1 &= 0.203 && \pm \; 1.580, \\
\hat{\beta}_2 &= -0.0972 && \pm \; 0.0417, \\
\hat{\beta}_3 &= -0.114 && \pm \; 0.104, \\
\hat{\beta}_4 &= 0.00504 && \pm \; 0.00434, \\
\hat{\beta}_5 &= -0.0112 && \pm \; 0.0052,
\end{aligned}$$

Calculate the modelled probability of chosing a BEV in the first two situations.

(g) Show that only the price and the charging time are significant factors for the decision.

(h) In order to check whether the full model is significantly more predictive than the simplified model

$$V_i = +\beta_2 C_i + \beta_5 T_1 \delta_{i1}$$

perform a LR test on these two models. In doing so, use the log-likelihoods $\tilde{L} = -30.1$ and $\tilde{L}^{\text{restr}} = -42.6$ of the calibrated full and restrained models, respectively.

(i) Most parameter estimates have no significance although the associated factors obviously matter. Explain why.

Quantiles $z_p = \Phi^{-1}(p)$ of the standardnormal distribution $\Phi(z)$

$p = 0.60$	0.70	0.80	0.90	0.95	0.975	0.990	0.995	0.999	0.9995
0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Quantiles $t_{n,p}$ of the Student t distribution with n degrees of freedom

n	$p = 0.60$	0.70	0.80	0.90	0.95	0.975	0.990	0.995	0.999	0.9995
1	0.325	0.727	1.376	3.078	6.315	12.706	31.821	63.657	318.31	636.62
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.153	3.707	5.208	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.154	4.587
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Quantiles $\chi^2_{n,p}$ of the χ^2 distribution with n degrees of freedom

n	$p = 0.9900$	0.9750	0.9500	0.9000	0.8000	0.5000	0.2000	0.1000	0.05000
1	6.635	5.034	3.821	2.706	1.656	0.4589	0.06540	0.01638	0.004230
2	9.210	7.378	5.991	4.605	3.219	1.386	0.4463	0.2107	0.1026
3	11.34	9.348	7.815	6.251	4.642	2.366	1.005	0.5843	0.3518
4	13.28	11.15	9.488	7.779	5.989	3.357	1.649	1.064	0.7106
5	15.09	12.83	11.07	9.236	7.289	4.351	2.343	1.610	1.155