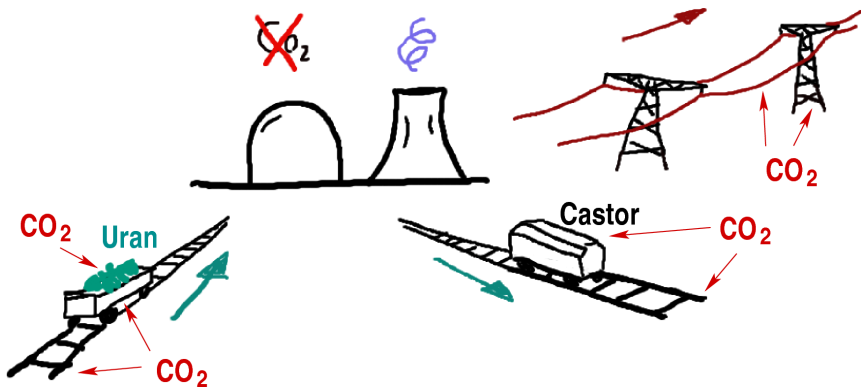


Chapter 5: Input-Output Models and Life-Cycle Assessment

- ▶ 5.1. Input-output model (IOM) of Leontief
- ▶ 5.2. Life-cycle assessment (LCA)
- ▶ 5.3. Combination: Econometric input-output LCA (EIO-LCA)

5.1. Input-output model (IOM) of Leontief: motivation



- ▶ Atomic power plants do not have any direct CO_2 emissions
- ▶ However, what are the *effective* emission considering all involved processes recursively?

Input-output model: problem statement

- ▶ In a modern economy, nearly everything is connected to “the rest” of the economy.
- ▶ *Wanted*: a quantitative description of the flows of material, services, and information between the different parts of an economy.
- ▶ The **input-output model (IOM) of Leontief** tackles this problem by making several assumptions:
 - ▶ Every material or service is associated with a certain **sector**.
 - ▶ To make all flows (kg, €, bytes, ...) commensurable, the common unit is a *monetary unit*, e.g., €.
 - ▶ The whole system is *linear* and *deterministic*: double input means double output. Particularly, there is no economy of scale.
 - ▶ The whole system is in the **steady state**, e.g., there are no temporal changes (constant supply and demand); storage (if applicable) is neither built up nor depleted.

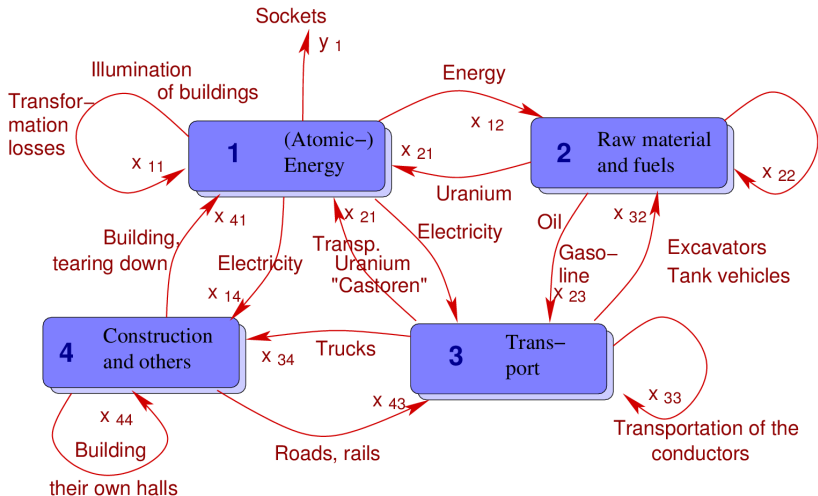
Specification of the IOM of Leontief

Linear, deterministic coupling of n sectors and an end consumer in the steady state:

$$x_i = y_i + \sum_{j=1}^n x_{ij} = y_i + \sum_{j=1}^n A_{ij}x_j.$$

- ▶ x_i : Total output of sector i in € or other monetary units per time unit.
- ▶ y_i : Flow of products/services of sector i to the end consumers (and to sectors that are not explicitly considered).
- ▶ x_{ij} : Flow from sector i to j : Sector j needs a supply x_{ij} from sector i to maintain the steady state and to ensure a constant supply y_j to the end consumer.
- ▶ $A_{ij} = x_{ij}/x_j$: IO coefficient reflecting linearity: In order to produce one unit, sector j needs A_{ij} units from all the other sectors i , including the own.

Visualisation of the flows generated by atomic power plants



Total production for a given consumer's supply

IOM equation in vector-matrix notation:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{x} + \mathbf{y}$$

- ▶ $\mathbf{x} = (x_1, x_2, \dots, x_n)^\top$ production vector
- ▶ $\mathbf{y} = (y_1, y_2, \dots, y_n)^\top$ supply vector
- ▶ $\mathbf{A} = (A_{ij}), i, j = 1 \dots n$ IOM coefficient matrix

Solving for \mathbf{x} by writing $(\mathbf{1} - \mathbf{A})\mathbf{x} = \mathbf{y}$:

$$\mathbf{x} = (\mathbf{1} - \mathbf{A})^{-1} \mathbf{y} \equiv \mathbf{B} \mathbf{y}$$

- ▶ $\mathbf{B} = (\mathbf{1} - \mathbf{A})^{-1}$ coefficient matrix of the final demand

Meaning of the matrix of the final demand **B**

- ▶ B_{ij} denotes the needed total production from sector i in order to deliver one unit of j to the end consumer (or the not considered sectors) in the steady state
- ▶ **B** includes all indirect effect *in an infinite recursion* as can be seen from the Taylor expansion:

$$\mathbf{B} = (\mathbf{1} - \mathbf{A})^{-1} = \mathbf{1} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \dots = \sum_{j=0}^{\infty} \mathbf{A}^j$$

Example:

$$B_{11} = 1 + A_{11} + \sum_{k=1}^2 A_{1k}A_{k1} + \sum_{k=1}^2 \sum_{l=1}^2 A_{1k}A_{kl}A_{l1} + \dots$$

Example: 1=transportation sector, 2=vehicle construction

- ▶ A_{11} : transport of bus/train/truck drivers to their workplace or driving vehicles to/from the depot or to/from services
- ▶ A_{12} : transport of vehicle construction workers to their workplace or of material needed to make the vehicles
- ▶ A_{21} : the continuous replacements of buses, trains and trucks needed to provide public or material transport of unchanged quality (steady state)
- ▶ A_{22} : the vehicle makers need vehicles themselves for their operations
- ▶ A_{11}^2 : In order to transport bus/train/truck drivers to their workplace one needs drivers as well
- ▶ $A_{11}A_{12}$: The transport of the vehicle construction workers to their workplace (A_{12}) induces additional traffic, hence additional need for operators/drivers (A_{11})
- ▶ $A_{12}A_{21}$: To manage the additional traffic caused by transporting the vehicle construction workers (A_{12}), the transportation sector needs additional vehicles (A_{21}) to maintain the steady state
- ▶ ...

Questions

- ? Argue that a national economy with sectors i satisfying $\sum_j A_{ij}x_j > x_i$ would not be sustainable or needs external help ("GDR").
- ? Give reasons why all A_{ij} and B_{ij} are ≥ 0 and $B_{ii} \geq 1$.
- ? Assume that the external demand y_i for products/services of sector i suddenly increases by one percent (e.g., driven by politics). Give a general expression for the percentaged increase of the total production of all the sectors in order to re-attain the steady state. Show that the result is independent from the price of an unit.
- ? Give some additional elements and concepts needed to make the IOM (i) dynamic, (ii) nonlinear reflecting the economy of scale.

5.2. Life-Cycle Assessment (LCA): Motivation

- ▶ The IOM reflects a *snapshot of all products* of a national economy in the *steady state*
- ▶ Sometimes, it is more instructive or relevant to consider *the total lifetime of a single product in a time dependent way* by assessing production, operation, and destruction/recycling of this product.
- ▶ This is formalized by the methods of **Life-Cycle Assessment (LCA)** (German: **Ökobilanz**).
- ▶ However, LCA only considers first-order indirect effects, e.g., CO₂ emissions caused by electric vehicles through the CO₂ footprint of electricity production
- ▶ The class of **Econometric Input-Output (EIO) LCA models** combines both approaches.

The standard LCA procedure

1. Define the life phases of the product in question:
 - ▶ production
 - ▶ operation/usage
 - ▶ destruction/recycling.
2. For each life phase, calculate the amount of needed materials/energy resulting in the **life-cycle inventory** \tilde{y}_j for product category j (the tilde denotes that the product is given in physical units such as kg or kWh rather than in €).
3. The total emissions e_i of pollutant i during the life time is obtained using the **emission factor matrix** **C**:

$$e_i = \sum_j C_{ij} \tilde{y}_j$$

where the emission factor C_{ij} gives the units of pollutant i caused by one unit of product j (including the production chain).

Example: Gasoline vehicle

Gasoline and Diesel vehicles are two examples of **internal combustion vehicles (ICV)**

1. Life-cycle inventory

- ▶ $\tilde{y}_1 = 800$ kg steel (900 kg at production time, 80 kg spare parts during lifetime, 20 % emission-neutral recycling contribution),
- ▶ $\tilde{y}_2 = 60$ kg aluminum (100 kg production, 40 % of it can be recycled without additional emissions)
- ▶ $\tilde{y}_3 = 100$ kg plastic
- ▶ $\tilde{y}_4 = 50$ kg rubber
- ▶ $\tilde{y}_5 = 36$ kg lead (three starter batteries à 12 kg)
- ▶ $\tilde{y}_6 = 15\,000$ l gasoline (250 000 km at 6 l/100 km during lifetime)

so we have

$$\tilde{\mathbf{y}} = (800 \text{ kg}, 60 \text{ kg}, 60 \text{ kg}, 60 \text{ kg}, 36 \text{ kg}, 15\,000 \text{ l})^T.$$

Example: Gasoline vehicle (ctnd)

2. Total CO₂ emissions

Defining e_1 to be the CO₂ emissions in kg (e_2 could be NO_x, e_3 PM and so on), we have

$$e_1 = \sum_{j=1}^6 C_{1j} \tilde{y}_j$$

with the row vector

$$C_{\text{CO}_2} = (C_{1j}) = (4, 30, 2, 2, 20, 2.7 \text{ kg/l}).$$

The last emission factor $C_{16} = C_{16}^{\text{w2t}} + C_{16}^{\text{t2w}}$ is the sum of two contributions:

- ▶ **Well-to-tank (w2t)** emissions of the production chain mining → transport to refinery → refining process → transport to the gas station: $C_{16}^{\text{w2t}} = 0.4 \text{ kg/l}$,
- ▶ **Tank-to-wheel (t2w)** emissions dictated by the chemistry during the actual combustion: $C_{16}^{\text{t2w}} = 2.3 \text{ kg/l}$ (gasoline) or $= 2.7 \text{ kg/l}$ (Diesel)

Example: Battery-electric vehicle (BEV)

- ▶ The **Life-cycle inventory** of steel, aluminum, rubber, plastic etc is comparable to that of the ICVs.
- ▶ The starter batteries are replaced by the Lithium driving batteries (2×300 kg) and the gasoline is replaced by the needed electrical energy, typically 20 kWh per 100 km:

$$\tilde{\mathbf{y}} = (800 \text{ kg}, 60 \text{ kg}, 60 \text{ kg}, 60 \text{ kg}, 600 \text{ kg}, 50\,000 \text{ kWh})^T.$$

- ▶ This leads to the new CO₂ emission factors vector

$$\mathbf{C}_{\text{CO}_2} = (4, 30, 2, 2, 20, 0.45 \text{ kg/kWh}).$$

- ▶ The Li driving batteries are expensive to produce and there is much controversy in estimating their overall emission factor C_{15}
- ▶ The energy emission factor is based, e.g., on the present (2019) German energy mix emitting 450 g CO₂ per kWh of electrical energy at the socket

Questions on LCA

- ? Is it possible to check, at a glance, whether the example BEV emits less CO₂ per km than the example ICV *when considering the driving phase alone*?
- ? How would you proceed to calculate the *break-even* mileage beyond which a BEV is more environmentally friendly (“green”) than the ICV?
- ? Give the two most important factors influencing the total LCA emissions of battery-electric vehicles.
- ? A common saying states that *the Sun does not issue invoices* nor does the production of electric energy by photovoltaic (PV) elements entail any direct CO₂ emissions. Discuss why PV energy still has a nonzero CO₂ footprint and how to calculate the PV CO₂ emission factor. Use LCA arguments and assume a steady state.

5.3. Econometric Input-Output LCA

See the [German script, Chapter 5.3](#).