Exam to the Lecture
Traffic Dynamics and Simulation
SS 2021
Solutions

Problem 1 (40 points)

(a) A time-continuous model.

(b) Free-flow acceleration parameters a: maximum acceleration (at zero speed), v_0: desired speed (free-flow acceleration is zero).

(c) Acceleration in the interaction regime: If smaller than the free-flow acceleration, the driver responds to the leader ("car-following regime"), otherwise, the leader is ignored (free-flow regime).

(d) In microscopic flow models, the fundamental diagram (FD) gives the homogeneous ($\frac{\partial}{\partial x} = 0$) steady-state ($\frac{\partial}{\partial t} = 0$) gap as a function of speed for identical vehicles (which can be transformed into a macroscopic FD such as flow as a function of density, afterwards). Homogeneous, i.e., no spatial changes, means same speed, $v_l = v$. Stationarity means no temporal changes at a certain location. Together with homogeneity, this also means no changes in the moving frame of micromodels, $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$.

(e) Sufficiently large gaps $\to$ the free-flow regime $\frac{\mathrm{d}v}{\mathrm{d}t} = a[1 - (v/v_0))^4]$ is active: With $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, we have $0 = a[1 - (v/v_0))^4]$ or $v = v_0$.

(f) The transition is abrupt because of the non-differentiable minimum function. In the steady-state $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $v_l = v$, we have at this transition

$$a\left[(1 - \left(\frac{v}{v_0}\right)^4\right] = a\left[1 - \left(\frac{s_0 + vT}{s}\right)^2\right] = 0$$

This means $v = v_0$ and $s = s_0 + vT = s_0 + v_0 T$. s_0: minimum gap at zero speed; T constant time gap leading to an increase of the steady-state gap with the speed.

(g) For $v < v_0$, there is no steady-state free flow, so the right term of the acceleration equation applies: $a[1 - ((s_0 + vT)/s)^2] = 0$ or $s = s_0 + vT$.

(h) With $l = 5\,\mathrm{m}$ and $s_0 = 3\,\mathrm{m}$, the *space headway* (distance between the fronts of identical vehicles) is $\Delta x_{\min} = l_{\text{eff}} = 8\,\mathrm{m}$, and the maximum density

$$\rho_{\max} = \frac{1}{l_{\text{eff}}} = \frac{1}{s_0 + l} = 125\,\mathrm{km}^{-1}$$

(i) In the steady-state interaction regime $s < s_0 + v_0 T$ or $\rho > \rho_c = 1/(l + s_0 + v_0 T)$, we have $v(s) = (s - s_0)/T$ or macroscopically with $s = 1/\rho - l$: $V(\rho) = v(1/\rho - l) = (1/\rho - l - s_0)/T$ while in the free regime, we simply have $V = v_0$. With the flow $Q = \rho V$, this leads to

$$Q(\rho) = \rho V(\rho) = \begin{cases} V_0 \rho & \rho \le \rho_c = \frac{1}{l + s_0 + v_0 T} \\ \frac{1 - \rho(l + s_0)}{T} & \rho > \rho_c \end{cases}$$

Unlike the FD of the "normal" IDM, this is the well-known tridiagonal fundamental diagram.

(j) Red traffic light corresponds to a leading virtal vehicle at speed $v_l = 0$. Approach with $v = v_0$, i.e. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ sufficiently far away. The critical gap is reached if the interacting part results in zero acceleration as well (with rapidly increasing deceleration afterwards):

$$0 = a\left[1 - \left(\frac{s^*}{s}\right)^2\right]$$

or

$$s = s^* = s_0 + v_0 T + \frac{v_0^2}{2\sqrt{ab}}$$

This is the formula for the stopping distance ("Anhalteweg") for a reaction time equal to T and a constant braking deceleration equal to $\sqrt{ab}$ (and equal to b, if $a = b$)

Problem 2 (40 points)

(a) In stationary situations ($\frac{\partial}{\partial t} = 0$), the continuity equation parallel to the merging region is given by

$$0 = \frac{\partial Q^{\text{tot}}}{\partial x} = \frac{Q_{\text{rmp}}}{L_{\text{rmp}}}, \quad x \in [-L_{\text{rmp}}, 0]$$

A trivial integration gives

$$Q^{\text{tot}}(x) = \frac{Q_{\text{rmp}}}{L_{\text{rmp}}} x + C$$

with the integration constant C determined by

$$Q^{\text{tot}}(-L_{\text{rmp}}) = -Q_{\text{rmp}} + C = Q_{\text{in}}, \quad \Rightarrow \quad C = Q_{\text{in}} + Q_{\text{rmp}}$$

so

$$Q^{\text{tot}}(x) = Q_{\text{in}} + Q_{\text{rmp}} \left(1 + \frac{x}{L_{\text{rmp}}}\right)$$

(and, more generally, $Q^{\text{tot}} = Q_{\text{in}}$ for $x < -L_{\text{rmp}}$ and $Q^{\text{tot}} = Q_{\text{in}} + Q_{\text{rmp}}$ for $x \geq 0$)

(b) Critical density per lane:

$$\rho_c = \frac{1}{l_{\text{eff}} + v_0 T} = \frac{1}{50\,\text{m}} = 20\,\text{veh/km}$$

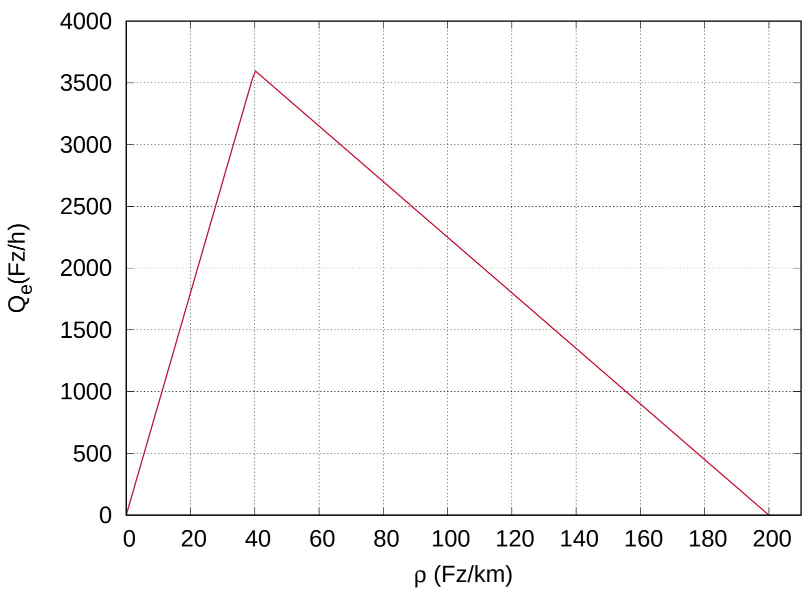
Maximum flow per lane:

$$Q_{\max} = V_0 \rho_c = 25\,\text{m/s}\ 0.02\,\text{veh/m} = 0.5\,\text{veh/s} = 1\,800\,\text{veh/h}$$

Capacity:

$$C = 2Q_{\max} = 3\,600\,\text{veh/h}$$

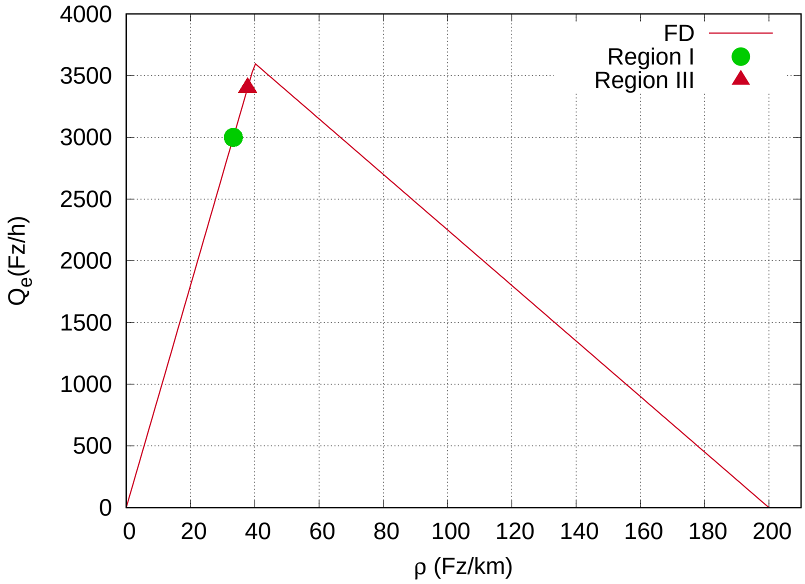
(c) Watch out that the diagram should be drawn for the *total* density and flow, i.e., sum of both lanes:



(d) Propagation velocities:

- Free flow: $w_{\text{free}} = v_0 = 25\,\text{m}$ (not any interaction in free flow!)
- Congested: $w_{\text{cong}} = w = -\frac{l_{\text{eff}}}{T} = -6.25\,\text{m/s}$

(e) Mainroad capacity $C = 3\,600\,\text{veh/h}$, sum $Q_{\text{in}} + Q_{\text{rmp}} = 3\,400\,\text{veh/h}$ is less than the capacity $\Rightarrow$ the traffic demand can be satisfied everywhere $\Rightarrow$ no congestions appear



(f) *Notice: The original problem statement asked for an impossible inflow of 4 200 veh/h. Therefore, this question and all the following dependent ones will not be evaluated. Extra points are earned for those spotting this error.*

With $Q_{\text{in}} = 3\,400\,\text{veh/h}$, the sum of the inflow and the ramp flow exceeds capacity. Consequently, a breakdown occurs as soon as the surge of the inflow has propagated to the effective location $x = 0$ of the on-ramp (negligible ramp length). With $w_{\text{free}} = v_0 = 25\,\text{m/s}$. This surge propagates the given 6-kilometer distance at a time of $6\,000\,\text{m}/25\,\text{m/s} = 240\,\text{s} = 4\,\text{min}$. So, the breakdown occurs at the time 16:04 at the location $x = 0$.[1]

(g)
- Region I, free inflow:

$$Q_1^{\text{tot}} = Q_{\text{in}} = 3\,400\,\text{veh/h}, \quad rho_1^{\text{tot}} = \frac{Q_{\text{in}}}{V_0} = 37.8\,\text{veh/km}$$

- Region II, congestion: Here we need the inverted FD for the congested branch which is always (!) given for the per-lane quantities: $\rho_{\text{cong}}(Q) = \frac{1-QT}{T}$

$$Q_2^{\text{tot}} = C - Q_{\text{rmp}} = 3\,200\,\text{veh/h}, \quad rho_2^{\text{tot}} = 2\rho_{\text{cong}}(Q_2^{\text{tot}}/2) = 57.8\,\text{veh/km}$$

This leads to a jam-front propagation velocity of

$$c_{12} = \frac{Q_2 - Q_1}{\rho_2 - \rho_1} = \frac{Q_2^{\text{tot}} - Q_1^{\text{tot}}}{\rho_2^{\text{tot}} - \rho_1^{\text{tot}}} = -2.778\,\text{m/s} = -10\,\text{km/h}$$

[1] In reality, it takes some time if the excess demand accumulates a sufficient number of vehicles, so, in relaity, the breakdown is somewhat later. However, this cannot be modelled with LWR models.

(h) Between 16:04 und 17:00, the upstream congestion front propagates at 10 km/h, so the maximum jam length is given by

$$x_{\max} = -c_{12} * 56 * 60\,\mathrm{s} = 9.33\,\mathrm{km}$$

The maximum jam time is determined by the speed inside the congestion:

$$T_{\max} = \frac{x_{\max}}{V_2} = \frac{x_{\max}\rho_2^{\mathrm{tot}}}{Q_2^{\mathrm{tot}}} = 607\,\mathrm{s}$$

(i) With the new inflow $Q_1^{\mathrm{tot}} = 3\,000\,\mathrm{veh/H}$, $rho_1^{\mathrm{tot}} = Q_1^{\mathrm{tot}}/V0 = 33.3\,\mathrm{veh/km}$, we now have

$$c_{12} = \frac{Q_2^{\mathrm{tot}} - Q_1^{\mathrm{tot}}}{\rho_2^{\mathrm{tot}} - \rho_1^{\mathrm{tot}}} = +2.27\,\mathrm{m/s}$$

and hence a dissolution time of

$$T_{\max} = \frac{x_{\max}}{c_{12}} = 4\,107\,\mathrm{s}$$

Although the excess supply of 200 veh/h after 17:00 corresponds to the excess demand before, the dissolution time of more than 68 min is longer than the buildup time of 56 min since, at dissolution time, we have less vehicles in the region $x \in -x_{\max}, 0]$ than at breakdown time since the inflow density decreased from 37.8 veh/km to 33.3 veh/km. The excess time for the dissolution com[pared to the buildup just corresponds to the time of removing $x_{\max} * 4.44\,\mathrm{veh/km}$ at a rate of the excess supply of 200 veh/h.

Problem 3 (25 points)

(a) A slow vehicle changes from another lane to the considered lane at time $t = 0$ and changes to a further (or the original) lane at 40 s effectively forming a *moving bottleneck* in between.

(b)
- Upstream region $x < 0$: $\rho = 5/600\,\mathrm{veh/m} = 8.3\,\mathrm{veh/km}$, $Q = 5/20\,\mathrm{veh/s} = 900\,\mathrm{veh/h}$, $v = 600\,\mathrm{m}/20\,\mathrm{s} = 30\,\mathrm{m/s} = 108\,\mathrm{km/h}$
- Congested region $x < 0$: $Q = 7/20\,\mathrm{veh/s} = 1\,260\,\mathrm{veh/h}$, $v = 10\,\mathrm{m/s} = 36\,\mathrm{km/h}$, $\rho = Q/V = 35\,\mathrm{veh/km}$
- Outflow region $x < 0$: $Q = 8/20\,\mathrm{veh/s} = 1440\,\mathrm{veh/h}$, $v = 30\,\mathrm{m/s} = 108\,\mathrm{km/h}$, $\rho = Q/V = 13.3\,\mathrm{veh/km}$

(c)
- Upstream jam front: $c_{\text{up}} \approx 300\,\mathrm{m}/70\,\mathrm{s}$
- Downstream front jam-empty: $c_{\text{down},1} = 10\,\mathrm{m/s}$ (the speed of the moving bottleneck)
- Downstream front jam-outflow: $c_{\text{down},2} = -50\,\mathrm{m}/40\,\mathrm{s}$

Of course, all propagation velocities can also be calculated using the shock-wave formula and the results of (b).

(d) Braking time $T_{\text{brake}} = 10\,\mathrm{s}$, speed change: $\Delta v = 10\,\mathrm{m/s} - 30\,\mathrm{m/s}$. Hence, the braking deceleration is given by

$$b = -\frac{\Delta v}{T_{\text{brake}}} = 2\,\mathrm{m/s^2}$$

Problem 4 (15 points)

(a) Possible reasons for the observed scattering of the flow-density data in spite of identical drivers and vehicles:
1. Traffic flow instabilities
2. Bias in estimating the density via the flow divided by the time mean speed

(b)
1. Heterogeneities of the vehicle-driver population
2. Even one and the same driver does not drive "like a machine" and instead shows all sorts of fluctuations in the driving style