Immatriculation nummer: (please also write it on all your work sheets)

Exam to the Lecture
Traffic Dynamics and Simulation
SS 2025

Total 120 points

Problem 1 (20 points)

(a) Given are the traffic flow data categories in following table and the use cases (i) to (vi). Indicate by an ✓ mark all data types which are applicable for each of the use cases (more than one category may be applicable for a given case):

(i) Determining the speed, at least in a biased way

(ii) Traffic state estimation & navigation

(iii) Adaptive control of a traffic light system at intersections

(iv) Estimating the density if the average vehicle length is known

(v) Unbiased density estimation

(vi) Unbiased speed estimation

Hint: Loop detectors can measure the passage time and the time of occupancy over the detector while simple counting detectors only measure the passage time.

Data type	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Single counting detector						
Single loop detector						
Two single counting detectors at a distance						
Double-loop detector						
Floating-car data						
Trajectory data						

(b) Explain the difference between the Eulerian and Lagrangian reference frame. What is the typically used reference frame for (i) micoscopic (ii) macroscopic models?

(c) At the end of a stationary downstream congestion front, a local speed $V = 10\,\mathrm{m/s}$ and a speed gradient $\frac{\partial V}{\partial x} = 0.1\,\mathrm{s}^{-1}$ is observed. Give the average acceleration of a vehicle passing this location.

Problem 2 (50 points)

Given is following acceleration equation for a car-following model as a function of the gap s, the speed v, and the leader's speed v_l:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = f = \min\left[a\left(1-\frac{v}{v_0}\right),\ a\left(1-\frac{s^*(v,v_l)}{s}\right)\right], \quad s^*(v,v_l) = s_0 + vT + \frac{v(v-v_l)}{2\sqrt{ab}}$$

For the numerical questions, assume the parameter values $v_0 = 15\,\mathrm{m/s}$, $T = 2\,\mathrm{s}$, $s_0 = 3\,\mathrm{m}$, $a = 2\,\mathrm{m/s^2}$, $b = 2\,\mathrm{m/s^2}$, and a vehicle length $l_{\mathrm{veh}} = 7\,\mathrm{m}$.

(a) Describe the workings of this model: its inputs, its output and its model parameters.

(b) Justify whether its acceleration function is (i) smooth differentiable with respect to the inputs, (ii) continuous but not everywhere differentiable, (iii) or discontinuous.

(c) Describe the meaning of the model parameters. Is the parameterisation appropriate for city or freeway traffic?

(d) When deriving the fundamental diagram (homogeneous steady-state relation), one has to set $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ and $v_l = v$. Why? Discuss this by referring to the definition of the fundamental diagram.

(e) Give the steady-state speed $v_e(s)$ as a function of the gap $s \geq s_0$. Distinguish the noninteracting case where the steady-state speed $v_e = v_0$ and the interacting case $v_e < v_0$.

(f) Derive the fundamental diagram $Q(\rho)$ using the definitions $\rho = 1/(l_{\mathrm{veh}} + s)$ and $Q = \rho v$ for the density and flow, respectively. Also give the capacity and the density at capacity for the given parameterisation.

(g) A driver approaches a red traffic light at the free-flow speed. At which critical distance s_{br} from the traffic light this driver will begin to brake? Is the braking deceleration abrupt or does it increase smoothly? Give the general result as a function of the model parameters and identify this distance as the sum of the minimum gap, the distance covered during the reaction time, and the braking distance for a constant deceleration. Associate the reaction time and the braking deceleration with model parameters. Give the numerical value for s_{br}.

(h) Show that the expressions for the steady-state acceleration sensitivities f_v and f_l with respect to speed and leading speed are given by

$$f_v = \left(\frac{\partial f}{\partial v}\right)_{\mathrm{e}} = \begin{cases} -\frac{2aT+\sqrt{\frac{a}{b}}v}{2s_e} & v < v_0 \\ -\frac{a}{v_0} & v \geq v_0 \end{cases}, \quad f_l = \left(\frac{\partial f}{\partial v_l}\right)_{\mathrm{e}} = \begin{cases} \sqrt{\frac{a}{b}}\frac{v}{2s_e} & v < v_0 \\ 0 & v \geq v_0 \end{cases},$$

respectively, where $s_e = s_0 + vT$. Do these sensitivities satisfy the plausibility conditions for microscopic models?

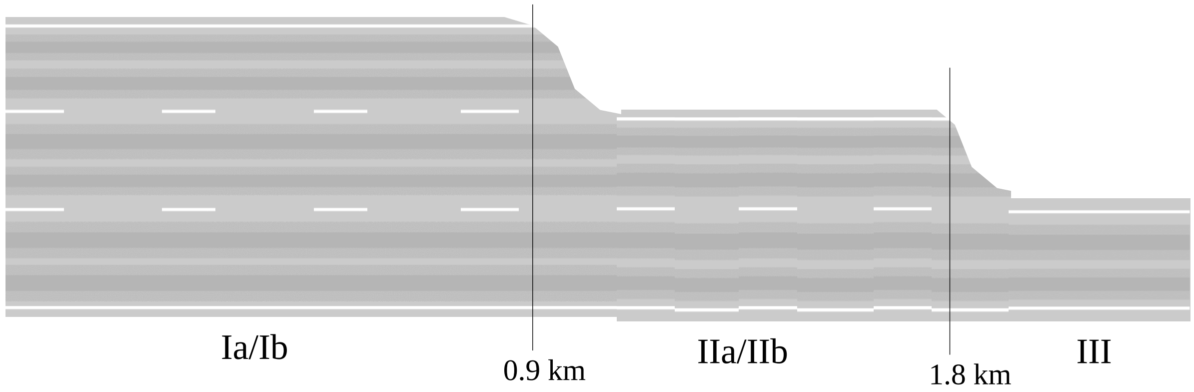
(i) Make a stability analysis for the steady state using the stability condition

$$v_e'(s) \leq \frac{1}{2}(f_l - f_v).$$

Distinguish the case $v = v_0$ (free traffic, $s_e \geq s_0 + v_0 T$) and $0 \leq v < v_0$ (interacting, $s_0 \leq s_e < s_0 + v_0 T$) and give the speed region of instability for the given parameters.

Problem 3 (50 points)

Given is a two-stage lane drop from three to two to one lane in a city traffic context (no traffic lights) according to following sketch



Traffic flow is modeled with an LWR model with a triangular fundamental diagram with the parameters

$$V_0 = 15\,\mathrm{m/s}, \quad T = 1.5\,\mathrm{s}. \quad l_{\text{eff}} = \frac{1}{\rho_{\max}} = 7.5\,\mathrm{m}.$$

(a) calculate the capacity per lane $Q_{\max}$, the density ρ_c at capacity, and the wave velocity w.

(b) The initial total traffic demand at $x = 0$ is given by $Q_{\text{in}} = 1\,500\,\mathrm{veh/h}$. Show that this does not cause a traffic breakdown. Calculate the speeds, flows, and densities in all three regions.

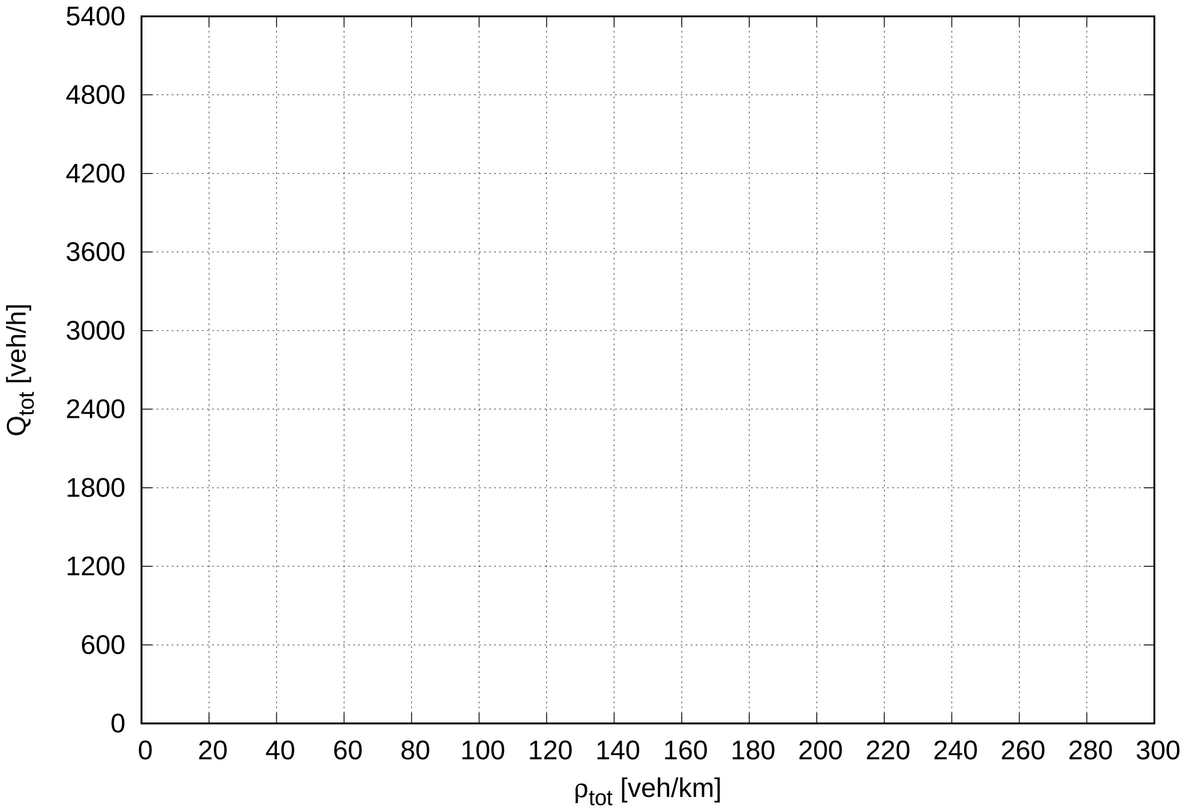
(c) At the begin of the rush hour ($t = 0$), the traffic demand suddenly increases to $2\,400\,\mathrm{veh/h}$. When and at which location a traffic breakdown is expected? Justify your answer.

(d) Now assume that the breakdown occurs at the two-to-one lane drop. Calculate flows and densities per lane for the free and congested states of Region II (IIa and IIb, respectively) and the shockwave velocity of the IIa $\rightarrow$ IIb transition.

(e) What happens once the shockwave reaches the location of the three-to-two lane drop?

(f) Now assume that the congestion propagates into Region I forming a free and congested part (Ia and Ib, respectively). Justify, using information propagation arguments, that the flow in Region Ib (congested) is given by $600\,\mathrm{veh/h}$ per lane while it is $800\,\mathrm{veh/h}$ per lane in the free Region Ia. Calculate the shockwave velocity at the transition Ia-Ib.

(g) Draw the states Ia, Ib, IIa, IIb, and III in the fundamental diagram of the total flows and densities at the next page. Also visualize the shockwave propagation velocities at the IA-Ib and IIa-IIb transitions.


5400
4800
4200
3600
3000
2400
1800
1200
600
0
Q_{tot} [veh/h]
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
ρ_{tot} [veh/km]