Exam to the Lecture
Traffic Dynamics and Simulation
SS 2024
Solutions

Total 120 points

Problem 1 (40 points)

(a) Signalized intersections are expected to be a little bit downstream of the vehicles stopping without a leader, i.e., at about $x_1 = 50\,\text{m}$ and $x_2 = 250\,\text{m}$ (and possibly at $x_3 = 355\,\text{m}$). The red phase starts when the first leader about to stop behind the traffic light *begins* to decelerate and ends a little bit before the first vehicle starts to accelerate (reaction time), i.e., 317 s - 350 s at x_1 and 317 s - 378 s at x_2 (and 310 s - 335 s at x_3 but the red phase may have started earlier)

(b) Explain the within-section beginnings and endings of some trajectories using the information that the displayed trajectories are for lane 2:

- beginning trajectories: lane changes 3-2 and 1-2
- ending trajectories: lane changes 2-3 and 2-1

(c) Calibration of trajectory 528:

- Time gap T: following episode 360 s-370 s.
- Minimum gap s_0: gap when stopped around 380 s (besides the trajectories, the vehicle lengths are needed for this as well).
- Comfortable deceleration b: deceleration phase 340 s-350 s but *not* the deceleration episode around 370 s: In the latter, the value of the true comfortable deceleration for the driver of this trajectory is just bounded from below by the deceleration of the leaders – without leaders, this driver may decelerate later implying a higher deceleration b.
- Desired speed v_0 and acceleration parameter a are not identified because the desired speed is never reached (this driver is always in the interacting regime), so v_0 can take on any value equal to or higher than the maximum observed speed. Likewise, a can be any value that is equal or greater than the leader's acceleration.

(d) Parameter identification:

- Desired speed v_0: a free-driving period at constant speed is needed: trajectory 487 at $t \le 310\,\text{s}$. It is unclear if trajectory 509 satisfies this criterion or if its acceleration phase around 355 s is directly followed by a deceleration phase without reaching v_0 (indicating this trajectory will give full marks)
- Acceleration a. Free accelerations are observed for trajectory 509 around 355 s and for trajectory 487 around 380 s (giving one trajectory will give full marks).

(e) Fundamental diagram (FD): With trajectory data, an unbiased estimation of speed, density, and flow values is possible if suitable traffic states exist. Any three out of the following four estimates define the triangular fundamental diagram (FD):

- Maximum density (waiting queue at $t = 380\,\mathrm{s}$):

$$\rho_{\max} = 1/l_{\text{eff}} = \frac{8}{60\,\mathrm{m}} = 133\,\mathrm{veh/km},$$

- wave velocity (sequence of the first eight starting vehicles between 380 s and 390 s):

$$w = \frac{-60\,\mathrm{m}}{10\,\mathrm{s}} = -21.6\,\mathrm{km/h},$$

- outflow from a queue or jam $Q_{\text{out}} \le Q_{\max}$ (the equality is only valid once V_0 is reached) by counting the vehicles passing $x = 250\,\mathrm{m}$ after 380 s:

$$Q_{\max} \ge \frac{10\,\mathrm{veh}}{20\,\mathrm{s}} = 1\,800\,\mathrm{veh/h},$$

- desired=maximum speed v_0: The average maximum speed of the trajectories for $t < 320\,\mathrm{s}$ (taking the fast vehicle 487 would lead to a bias: V_0 is the *average* desired speed over all trajectories):

$$V_0 = 20\,\mathrm{m/s} = 72\,\mathrm{km/h}.$$

Since the calculation of the outflow is error prone and the estimation above is biased towards lower values (the outflow is only equal to $Q_{\max}$ once it reaches the desired speed), we use V_0, w, and $\rho_{\max}$ to calculate the FD as

$$Q(\rho) = \min\left(V_0\rho, \frac{1}{T}\left(1 - \frac{\rho}{\rho_{\max}}\right)\right) = \min\left(V_0\rho, w(\rho - \rho_{\max})\right).$$

Here, we expressed the FD parameter T by observable values via $T = -(\rho_{\max} w)^{-1}$.

Side information: Using the estimates as above, the implied maximum flow (equating the two terms of the minimum function) is given by

$$Q_{\max}^{\text{calc}} = \frac{V_0 \rho_{\max}}{1 - \frac{V_0}{w}} = 0.61\,\mathrm{veh/s} = 2210\,\mathrm{veh/h}$$

confirming the suspicion that estimating $Q_{\max}$ by the number of crossing vehicles gives a bias towards lower values.

Problem 2 (40 points)

(a) In macroscopic models of second order, both the local density $\rho(x,t)$ and the speed $V(x,t)$ are independent dynamical variables while the flow is given by $Q = \rho V$ (we can equivalently, chose the dynamical variables ρ and Q with the speed given by $V = Q/\rho$). In contrast, in first-order models, the speed is not an independent dynamic variable but given in terms of the local density by the static *fundamental* relation $V(x,t) = V_e(\rho(x,t))$

(b) Since first-order or LWR models have a static speed, no flow instabilities are possible, only a single transition free → congested if the demand exceeds the supply, i.e., the local capacity. Suitably parameterized second-order models show string instability, i.e., they can describe growing traffic waves and flow instabilities.

(c) The dynamic speed equation of the Kerner-Konhäuser model is given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} \equiv \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e(\rho) - V}{\tau} - \frac{c_0^2}{\rho}\frac{\partial \rho}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 V}{\partial x^2}$$

Multiplying this equation by τ gives

$$\tau\frac{\partial V}{\partial t} + \tau V\frac{\partial V}{\partial x} = V_e(\rho) - V - \frac{\tau c_0^2}{\rho}\frac{\partial \rho}{\partial x} + \frac{\tau\mu}{\rho}\frac{\partial^2 V}{\partial x^2}$$

For continuous densities and speeds, all partial derivatives of $\rho(x,t)$ and $V(x,t)$ are finite so, for $\tau \to 0$, all terms containing τ tend to zero resulting in

$$V_e(\rho) - V = 0$$

which is precisely the fixed speed definition $V = V_e(\rho)$ characterizing LWR models.

(d) Which models suit best? We abbreviate

- MIC=microscopic models,
- LWR=Lighthill-Whitham-Richards models (first-order macroscopic models),
- MAC2 (second-order macroscopic models)

and check them for following tasks:

(i) *Model the dynamics (including traffic breakdown) at an off-ramp bottleneck*: MIC. While also LWR can model breakdowns and MAC2 even traffic instabilities, both models treat *off*-ramps as a local *increase* of capacity (since part of the traffic leaves the road), i.e., "anti-bottlenecks" which never lead to a breakdown. In contrast, when adding lane changing to microscopic models, the associated perturbation may trigger a breakdown even if there is *less* traffic after the off-ramp.

(ii) *determine if it is likely that on-ramp bottlenecks on a freeway will lead to traffic jams in the vacation season*: LWR. Both MIC and MAC2 are "overkill" in this situation since only the comparison of demand (vacation traffic) and supply (identifying the bottlenecks) is needed for this task.

(iii) *model the effect of empty and full trucks at gradient sections*: MIC. Macroscopic models are only possible if extending them to at least two vehicle classes and two lanes resulting in four interacting partial densities (and additionally four speed fields for MAC2) leading to very coomplex models.

(iv) *creating responsive surrounding traffic in driving simulators*: Clearly MIC. The people driving in simulators want to see vehicles, not local densities.

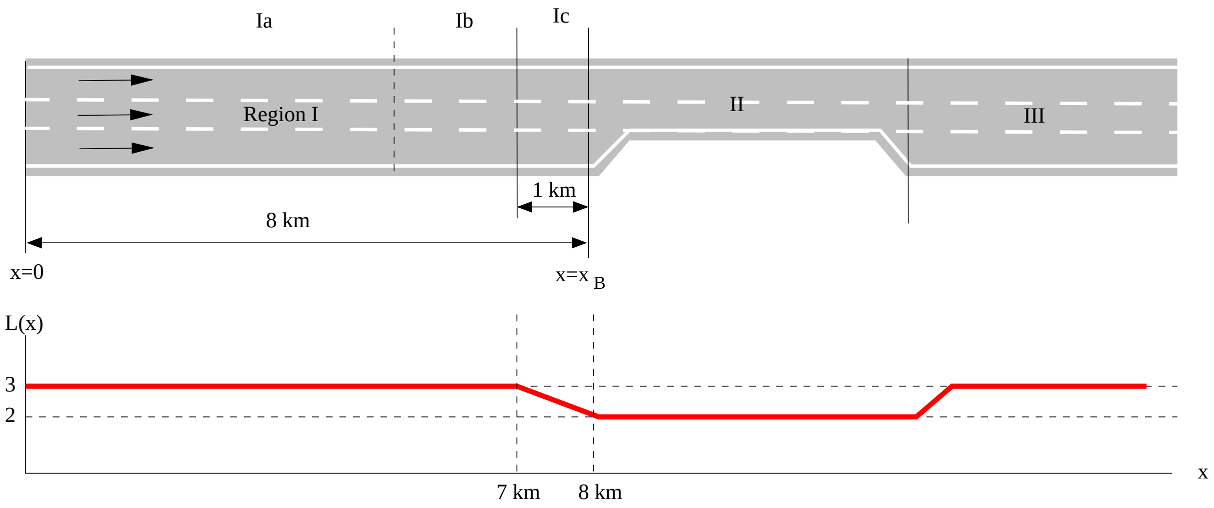
(v) *Determine instabilities to traffic waves as a function of the density and the average driving style*: MAC2. MIC would be possible as well but would be "overkill".

(e) Full Gipps model with the model parameters desired speed v_0, acceleration a, time gap T, minimum gap s_0, safety cushion ϑ, assumed own deceleration b, and assued leader's deceleration b_l. Associate following driving characteristics with high or low values of these parameters (normal values or irrelevant parameters need not to be mentioned):

- fast: v_0 high,
- quickly accelerating: a high,
- aggressive: v_0, a, and b high; s_0 and T low,
- anticipative/experienced: b low, $b_l > b$, a rather high (leads to responsive driving),
- safety oriented: T and s_0 high, $b_l > b$, v_0 rather low.

Problem 3 (40 points)

Given is following freeway road section with a lane-closing bottleneck and also the number of the real-valued effective number $L(x)$ of lanes as a function of the position x:



(a) If a lane closing is ahead, the drivers on the lane to be closed will change to the other lanes well in advance (several 100 m) so hardly anybody wants to drive on this lane near the lane closing. This means, the effective capacity of this lane is continuously reduced from $Q_{\max}$ to zero at or very near the closing point. For the whole diectional road, we therefore have a location dependent capacity $C(x) = L(x)Q_{\max}$ with the fractional lane number $L(x)$ continuously decreasing from 3 to 2 when getting closer to the lane closing.

(b) The fundamental diagram is tridiagonal and given by

$$Q(\rho) = \max\left(V_0\rho, \frac{1}{T}(1 - \frac{\rho}{\rho_{\max}})\right).$$

In terms of the observed per-lane quantities $Q_{\max} = 1\,800$ veh/h $= 0.5$ veh/s, wave speed $w = -18$ km/h $= -5$ m/s, and desired speed $V_0 = 90$ km/h $= 25$ m/s, we obtain the density at capacity $\rho_c = Q_{\max}/V_0 = 20$ veh/km, the maximum density $\rho_{\max} = \rho_c - Q_{\max}/w = 120$ veh/km (using $Q(\rho_c) = Q_{\max}$ and $Q'(\rho) = w$ for $\rho > \rho_c$), and the implied time gap $T = (\rho_{\max} w)^{-1} = 1.67$ s.

(c) The bottleneck capacity is given by $C_B = 2Q_{\max} = 3\,600$ veh/h which is above $Q_{\text{in}} = 2\,700$ veh/h. Hence, no traffic breakdown.

(i) Total quantities: Since there is no congestion and we have a constant desired speed, the total quantities are unchanged throughout the considered road section and given by

$$Q^{\text{tot}}(x) = Q_{\text{in}} = 2\,700 \text{ veh/h}, \quad \rho^{\text{tot}}(x) = \frac{Q^{\text{tot}}(x)}{V_0} = 30 \text{ veh/km}.$$

(ii) Since both flow and densities are extensive quantities (they increase with the number of vehicles), the total quantities are equal to the per-lane quantities times the number of lanes. Hence

$$Q(x) = \frac{Q^{\text{tot}}(x)}{L(x)} = \frac{Q_{\text{in}}}{L(x)}, \quad \rho(x) = \frac{\rho^{\text{tot}}(x)}{L(x)} = \frac{Q_{\text{in}}}{V_0 L(x)}.$$

(d) Traffic will break down at a position where the sudden demand surge to $Q_{\rm in} = 4\,500$ veh/h will exceed the local capacity, for the first time. Since $Q_{\rm in} = 2.5 Q_{\rm max}$ corresponds to the capacity for 2.5 fractional lanes and the fractional lane number decreases linearly from 3 to 2 between 7 km and 8 km, the breakdown location is at $x_{\rm bd} = 7.5$ km. The flow surge for $x < x_{\rm bd}$ can be considered as a shockwave in the free-flow regime propagating at a velocity equal to $V_0 = 90$ km/h, or 1.5 km per minute. Hence, this shock reaches the breakdown point at $t = x_{\rm bd}/V_0 = 5$ minutes.

(e) Info: After a short time, the downstream front of the traffic jam forms at the position where the minimum local capacity is reached for the first time at $x = x_{\rm B} = 8$ km

- Region 1a (free traffic):

$$
\begin{aligned}
Q_{\rm 1a} &= \frac{Q_{\rm in}}{3} = 1\,500\ \text{veh/h/lane},\\
\rho_{\rm 1a} &= \frac{Q_{\rm in}}{V_0} = 16.7\ \text{veh/km/lane},\\
V_{\rm 1a} &= V_0 = 90\ \text{km/h}.
\end{aligned}
$$

- Region 1b (congested traffic):

$$
\begin{aligned}
Q_{\rm 1b} &= C_{\rm bottl} 3 = 1\,200\ \text{veh/h/lane},\\
\rho_{\rm 1b} &= \rho_{\rm cong}(Q_{\rm 1b}) = \rho_{\rm max}(1 - Q_{\rm 1b} T) = 53.3\ \text{veh/km/lane},\\
V_{\rm 1b} &= \frac{Q_{\rm 1b}}{\rho_{\rm 1b}} = 6.25\ \text{m/s} = 22.5\ \text{km/h}.
\end{aligned}
$$

(f) Shock-wave formula:

$$
c = \frac{Q_{\rm 1b} - Q_{\rm 1a}}{\rho_{\rm 1b} - \rho_{\rm 1a}} = -2.27\ \text{m/s} = -8.18\ \text{km/h}.
$$