Name:	First name:	Matrikel number:

Exam to the Lecture
Traffic Dynamics and Simulation
SS 2021

Total 120 points

Problem 1 (40 points)

Given is following acceleration equation for a car-following model as a function of the gap s, the speed v, and the leader's speed v_l:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \min\left\{ a\left[1-\left(\frac{v}{v_0}\right)^4\right],\ a\left[1-\left(\frac{s}{s^*}\right)^4\right]\right\}, \quad s^* = s_0 + vT + \frac{v(v-v_l)}{\sqrt{2ab}}$$

(a) Is this a time-continuous or a time-discrete car-following model?

(b) Discuss, in a few words, the first term $a(1-(v/v_0))^4$ and give the meaning of the model parameters a and v_0.

(c) Discuss the second term $a(1-(s/s^*)^2)$.

(d) When deriving the fundamental diagram (homogeneous steady-state relation), one has to set $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ and $v_l = v$. Why? Discuss by referring to the definition of the fundamental diagram.

(e) Give the steady-state speed for sufficiently large gaps where no interaction occurs.

(f) Show that, in the steady state, the transition from free traffic to the congested state is abrupt and at a gap $s_0 + v_0T$. Discuss the model parameters s_0 and T

(g) Give the steady-state gap as a function of the speed for speeds less than the desired speed.

(h) Given is a traffic flow of identical drivers and vehicles of 5 m length. Give the maximum density for $s_0 = 3\,\mathrm{m}$

(i) Give the fundamental diagram $Q_e(\rho)$ of this model. Assume a free-flow speed of v_0 and a steady-state gap $s_e(v) = s_0 + vT$ for speeds $v < v_0$.

(i) A driver driving according to this model approaches a red traffic light at the free-flow speed. At which distance from the traffic light this driver begins to brake? Give the general result as a function of s_0, v_0, T, and b and identify this distance as the sum of the minimum gap, the distance covered during the reaction time, and the braking distance for a constant deceleration and associate the reaction time and the braking deceleration with model parameters.

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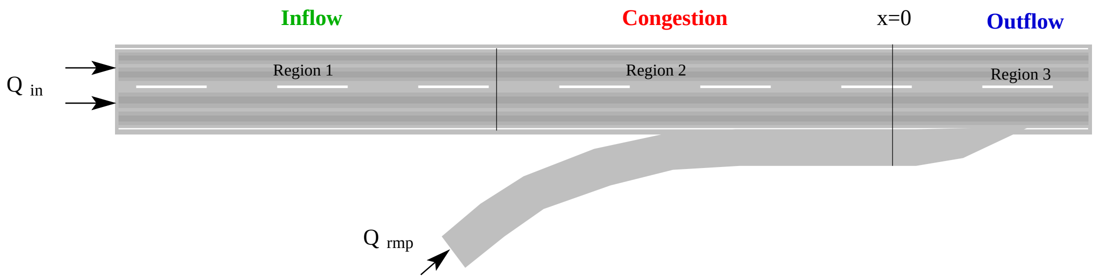
Problem 2 (40 points)

Given is a freeway road section with an on-ramp merging to the main road effectively at the location $x = 0$ according to following sketch (no jammed region 2 at the beginning):



For the analysis, assume a macroscopic LWR model and a triangular fundamental diagram with the parameters

$$V_0 = 25\,\mathrm{m/s}, \quad T = 1.6\,\mathrm{s}, \quad \rho_{\max} = \frac{1}{l_{\mathrm{eff}}} = 0.1\,\mathrm{m}^{-1}.$$

(a) Assuming a ramp of merging length l_{rmp}, the continuity equation of this situation for the total density and flow (summed over both lanes) can be written as

$$\frac{\mathrm{d}\rho^{\mathrm{tot}}}{\mathrm{d}t} + \frac{\mathrm{d}Q^{\mathrm{tot}}}{\mathrm{d}x} = \begin{cases} \frac{Q_{\mathrm{rmp}}}{L_{\mathrm{rmp}}} & x \in [-L_{\mathrm{rmp}}, 0], \\ 0 & \text{otherwise.} \end{cases}$$

Calculate for stationary situations ($\frac{\mathrm{d}}{\mathrm{d}t} = 0$ but of course $\frac{\mathrm{d}}{\mathrm{d}x} \neq 0$ in the merging region) the total main flow $Q^{\mathrm{tot}}(x)$ as a function of the constant total main inflow Q^{in} and the ramp flow Q_{rmp}.

(b) Calculate the numerical values for the critical density ρ_c per lane at the free-congested transition, the maximum flow per lane, and the main-road capacity.

(c) Draw the fundamental diagram for the total flows on the main road

(d) Calculate the propagation velocity of small perturbations in free flow and in congested traffic.

(e) The ramp flow is constant $Q_{\mathrm{rmp}} = 400\,\mathrm{veh/h}$ and, at the beginning of the analysis, the main inflow $Q_{\mathrm{in}} = 3\,000\,\mathrm{veh/h}$. Argue that these demands will not lead to a traffic breakdown (whatch out for the units!). Draw the total flows and densities of Region $\tilde{1}$ (upstream of the ramp) and Region 3 (downstream) into the fundamental diagram of Question (c).

(f) At 16:00 h, the main inflow as observed by a stationary detector 6 km upstream ($x = -6\,\mathrm{km}$) suddenly increases from $3\,000\,\mathrm{veh/h}$ to $4\,200\,\mathrm{veh/h}$ while the ramp flow remains constant at $400\,\mathrm{veh/h}$. Argue that this increase of the demand will lead to a breakdown. Determine the location and time of the breakdown. *Hints:* Here, you can assume a negligible ramp length. Watch out for the finite propagation velocity of flow changes in free flow.

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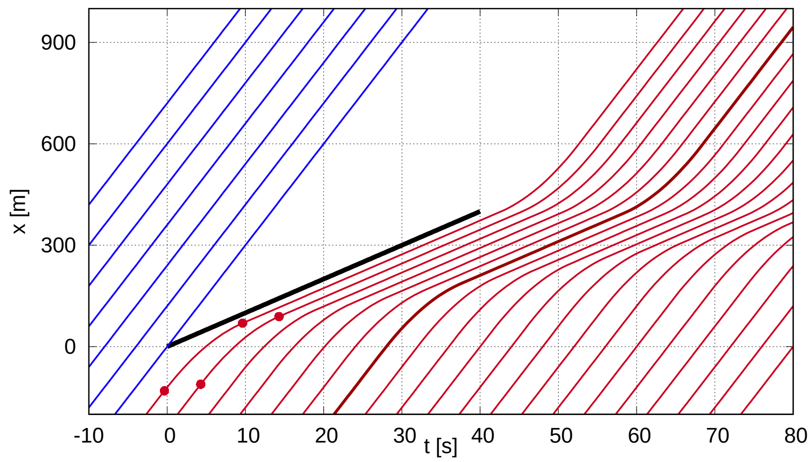
(g) Calculate the propagation velocity of the upstream front of the congestion resulting from the breakdown. Draw the traffic states of the Regions 1, 2, and 3 into the fundamental diagram of Question (c).

(h) At 17:00 h, the traffic flow immediately upstream of the transition free → jammed drops from 4 200 veh/h to 2 200 veh/h. Calculate the maximum length of the congestion and the loss of time for the most unhappy driver (arriving just at 17:00 h at the rear end of the congestion).

(i) Calculate the time at which the congestion dissolves assuming no further demand changes after 17:00. Motivate why the dissolution takes a longer time than the buildup although the excess supply after 17:00 is equal to the excess demand before.

Problem 3 (25 points)

Given is following trajectory diagram for one lane of a freeway:



(a) Which situation can lead to such trajectories? (Notice that the thick black line represents a vehicle as well).

(b) Determine density, speed, and flow for the three regions

- upstream, $x \leq 0$
- congested region
- outflow state ($t > 60$ s and $x > 600$ m).

(c) Determine the propagation velocities of the upstream and downstream transition zones of the congestion.

(d) Estimate the braking deceleration (the braking phase is between the two red closed circles).

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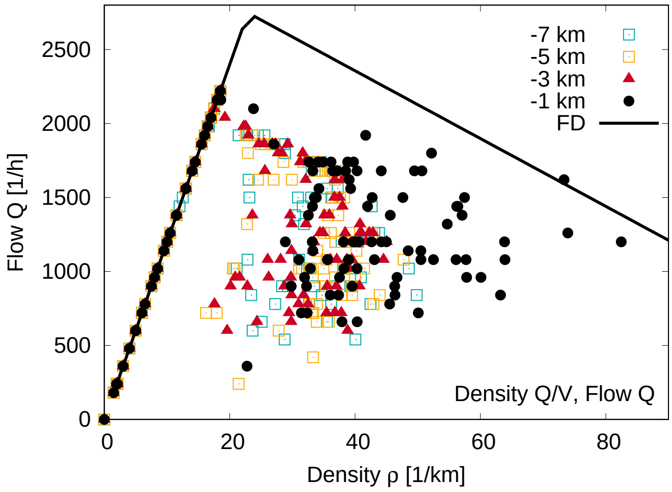
Problem 4 (15 points)

Given is a flow-density scatter plot obtained from virtual stationary detectors of a microscopic traffic flow simulation (symbols) together with the fundamental diagram of the simulated model.



(a) Obviously, the points of the scatter plot do not lie on the fundamental diagram although a deterministic micromodel with identical drivers and vehicles has been simulated. What are possible reasons?

(b) Discuss further irregularities or stochastic elements of real traffic that may also lead to the observed scattering.