Lecture 13: Dynamic Navigation

- 13.1 Load Balancing and Dynamical Navigation
- 13.2 Macroscopic Approach
- 13.3 Microscopic Approach

13.1 Load Balancing and Dynamical Navigation

- Inefficiencies in network usage possibly leading to breakdowns can be alleviated by **load balancing** a.k.a. **dynamic routing** using routes that are longer without traffic.

  Effects of dynamic routing: "Longer is sometimes shorter"

- Traditionally, dynamic routing was implemented by **variable message signs (VMS)**
- Nowadays, this has essentially replaced by client-side **dynamic navigation** by smartphones or built-in navigation

The Curse of Latency

The problem is that the actions of dynamic navigation have a *delayed* effect on the system itself. What looks like a cute shortcut at decision time may bring you in a jam at a bottleneck of the weak secondary network

Criteria for dynamic routing

Identify the diverging node A and merging node B for a possible detour, i.e., there are two routes from A to B. Which route should I take?

- **Realized travel time**: Take the route with the shortest travel time from A to B as recorded by the last vehicles on either route
- **Instantaneous travel time**: Calculate the travel times on either route r with the last recorded *local speeds* $V(x_r)$:

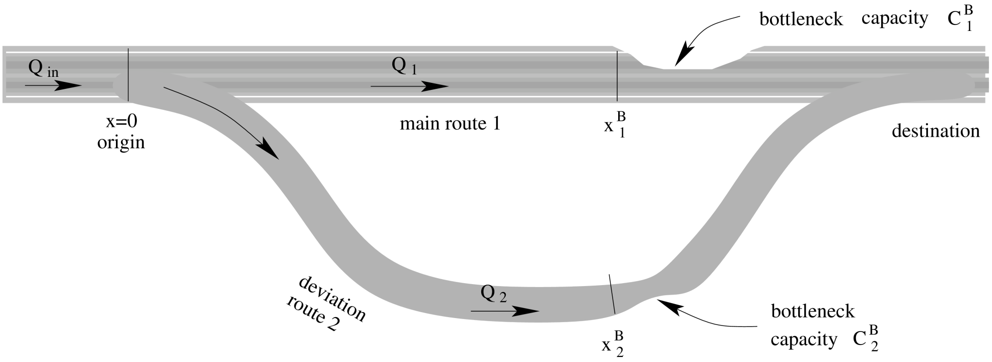
$$T_r(t) = \int_A^B \frac{1}{V(x_r, t)} \, \mathrm{d}x_r$$

- **Expected travel time**: Use traffic flow models to predict $V(x_r, t+\tau)$ for $\tau > t$ and then solve the following ODE for Point B:

$$T_r^{(E)}(t) = t_r(B), \quad \frac{\mathrm{d}t_r(x_r)}{\mathrm{d}x_r} = \frac{1}{V(x_r, t_r)}, \quad \text{IC:} \;\; t_r(A) = t$$

- Add **random time contributions** ϵ to reflect uncertainties or different data providers, introduce delays reflecting the finite response time of the information processing chain

13.2 Macroscopic Approach for a Simple Network



- Two routes of length L_1 (main route) and L_2 (detour). Point A at $x_r = 0$, point B at $x_r = L_r$
- The routes have capacities C_r and local bottlenecks with $C_r^{\text{B}} < C_r$
- We use a LWR model with a tridiagonal FD and specifications $V_{0r} = V_0$ and $w_r = w$ (just simplification, no loss of generality) and, of course maximum total flows equal to the capacities C_r
- Assume $Q_{\text{in}} < C_1$, so no congestion for $x_1 < 0$ or beyond point B

Macroscopic routing decision model

- Route utility is based on its instantaneous travel time T_r and an additive Gumbel distributed random part ϵ_r

$$U_r = U_r^{\text{det}} + \epsilon_r = \beta T_r + \epsilon_r, \quad \frac{1}{\beta_r} \approx \sigma_T = \text{stddev}(\epsilon_r)$$

- $\Rightarrow$ Logit model for the probability P_2 that, at an equipment level α, the deviation route 2 is used:

$$P_2 = \frac{\alpha e^{U_2^{\text{det}}}}{e^{U_1^{\text{det}}} + e^{U_2^{\text{det}}}}, \quad P_1 = 1 - P_2 \tag{1}$$

- The instantaneous travel times of the routes follows from the LWR model:

$$T_r(t) = \frac{L_r - l_r^{\text{jam}}(t)}{V_0} + \frac{l_r^{\text{jam}}(t)}{V_r^{\text{cong}}}, \tag{2}$$

  - Behind the respective bottlenecks at x_r^{B}, jams of length $l_r^{\text{jam}} = x_r^{\text{B}} - x_r^*$ may form
  - The congested speed follows from the tridiagonal FD

$$V_r^{\text{cong}} = V^{\text{cong}}(C_r^{\text{B}}) = \frac{C_r^{\text{B}} V_0 w}{V_0(C_r^{\text{B}} - C_r) + w C_r}$$

Dynamics of the upstream jam front locations x_r^*

This follows from the *shock-wave formula* together with the fixed propagation velocities V_0 and w of the tridiagonal FD:

$$\frac{\mathrm{d}x_r^*}{\mathrm{d}t} = \begin{cases} \frac{Q_r^{\text{free}}(x_r^*(t),t)-Q_r^{\text{cong}}}{Q_r^{\text{free}}(x_r^*(t),t)/V_0-\rho_r^{\text{cong}}} & \text{if } x_r^* < x_r^{\text{B}} \text{ or } Q_r^{\text{free}}(x_r^*(t),t) > Q_r^{\text{cong}} \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

- Free flow propagates from the diversion point A ($x=0$) with V_0 ($I_r = $ #lanes at route r)

$$Q_r^{\text{free}}(x^*,t) = Q_{\text{in}}/I_r \ \left(t - x^*/V_0\right) P_r\left(t - x^*/V_0\right)/I_r, \quad \rho_r^{\text{free}}(x^*,t) = \frac{Q_r^{\text{free}}(x^*,t)}{V_0}$$

- Congested flow propagates from the bottleneck at $x = x_r^{\text{B}}$ upstream at wave speed w (constant bottleneck capacity $\rightarrow$ no time dependence)

$$Q_r^{\text{cong}} = \frac{C_r^{\text{B}}}{I_r}, \quad \rho_r^{\text{cong}} = -\frac{1}{w}\left[Q_r^{\text{max}}\left(1 - w/V_0\right) - Q_r^{\text{cong}}\right],$$

- $\Rightarrow$ Eq. (3) with (1) and (2) is a set of delay-differential equations since Q_r^{free} depends on past times

Steady state and user equilibrium (UE)

Assume a two-route system with $T_{01} < T_{02}$ and $C_1 > C_2$ (Route 1 is the main route), furthermore $C_1^{\mathrm{B}} < Q_{\mathrm{in}} < C_1$ and $C_1^{\mathrm{B}} + C_2^{\mathrm{B}} > Q_{\mathrm{in}}$ (network can accomodate the demand but Route 2 is needed):

User equilibrium (UE)

$$T_1 = T_2 = T_{02} = T_{01} + l_1^{\mathrm{jam}} \left(\frac{1}{V_1^{\mathrm{cong}}} - \frac{1}{V_0} \right), \quad P_2^{\mathrm{E}} = \frac{Q_{\mathrm{in}} - C_1^{\mathrm{B}}}{Q_{\mathrm{in}}}$$

- A jam forms behind the bottleneck of route 1 until both travel times are equal, then, a fraction P_2 of vehicles are diverted
- Because of the delays, this stationary UE is rarely reached and route oscillations appear, particularly for large α
- Necessary condition: equipment level $\alpha > P_2^{\mathrm{E}} = (Q_{\mathrm{in}} - C_1^{\mathrm{B}})/Q_{\mathrm{in}}$
- For $\alpha > P_2$ and sufficient uncertainty (!) σ_T, a steady state between the UE and the system optimum can be reached

System optimum (SO)

The SO minimizes the overall travel times $P_1 T_1 + P_2 T_2$.

- Obviously, this is reached for the *same assignment* $P_2 = P_2^{\mathrm{E}}$ but without jams, i.e., the vehicles on route 2 drive longer
- In the deterministic case unstable
- With sufficient random time contributions of standard deviation $\approx \sigma_T$, the SO can be stabilized since, sometimes, ϵ_T overrides the deterministic disadvantage:

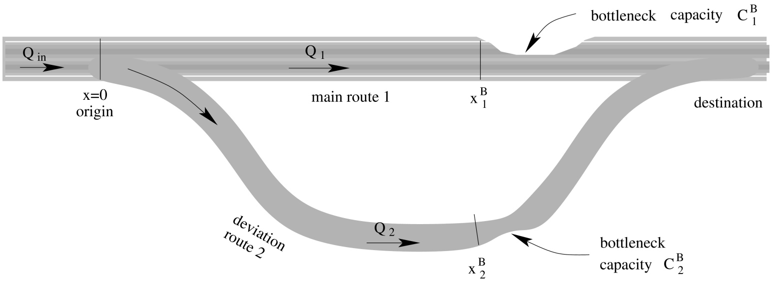
$$P_2 = \frac{\alpha}{e^{V_{01} - V_{02}} + 1} = \frac{\alpha}{e^{\frac{T_{02} - T_{01}}{\sigma_T}} + 1} \stackrel{!}{=} P_2^{\mathrm{E}}$$

so

$$\alpha = P_2^{\mathrm{E}} \left(e^{\frac{T_{02} - T_{01}}{\sigma_T}} + 1 \right).$$

Sometimes, uncertainty leads to stabilisation

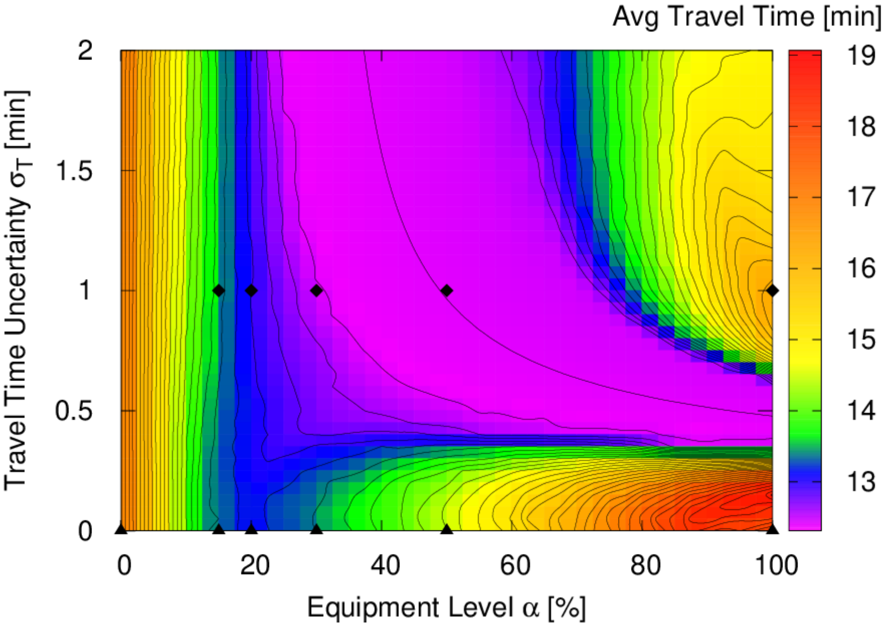
Simulation specification



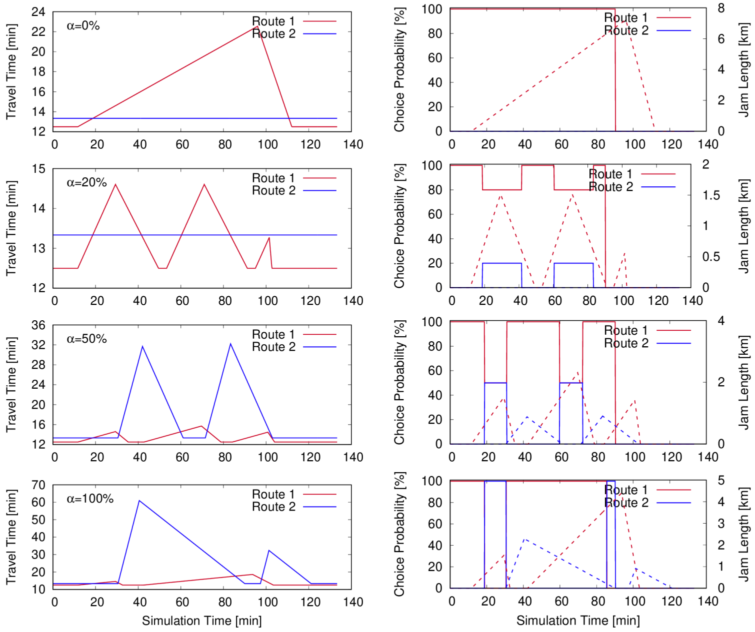
- Rush-hour inflow $Q_{\text{in}}(t) = \begin{cases} 5,400\,\text{veh/h} & 0 \le t \le 5,400\,\text{s}, \\ 0 & \text{otherwise.} \end{cases}$
- Capacities (both routes have $I = 3$ lanes outside of the bottlenecks) $C_1 = 6,480\,\text{veh/h},\ C_1^B = 4,860\,\text{veh/h},\ C_2 = 6,480\,\text{veh/h},\ C_2^B = 1,080\,\text{veh/h}$
- Route lengths $L_1 = 15\,\text{km}.\ L_2 = 16\,\text{km}$
- Bottleneck locations $x_1^{\text{B}} = x_2^{\text{B}} = 14\,\text{km}$
- LWR parameters

$$Q_{\text{max}} = \frac{C_i}{I_i} = 2\,160\,\text{veh/h}, \quad V_0 = 72\,\text{km/h}, \quad w = -18\,\text{km/h}$$

Averaged travel time for deterministic route choices (Logit parameter $\sigma_T = 0$)



Travel Times Fluctuate in the deterministic case!



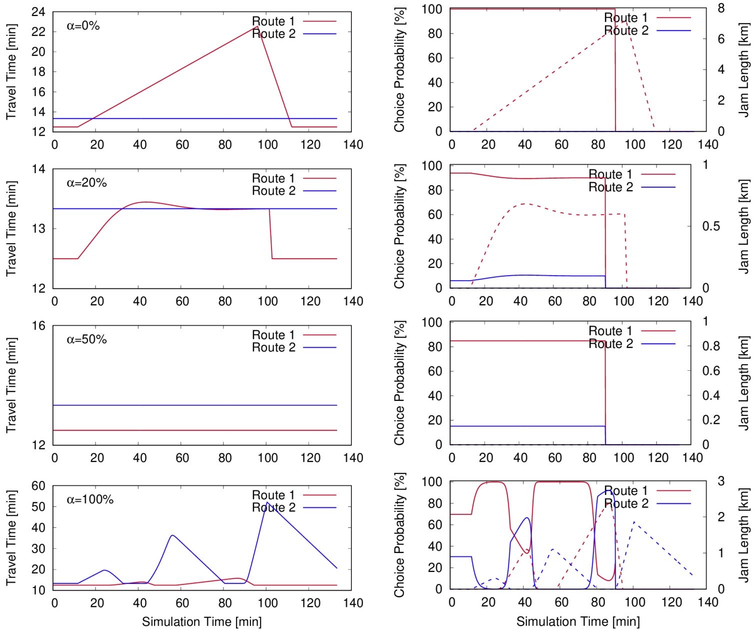
No equipped cars: Jam growth on R1 only stopped by end of rush hour

$\alpha = 20\,\%$: Oscillating jam on R1, but not growing

$\alpha = 50\,\%$: Massive routing on R2 leads to jams on both routes but particularly on R2

$\alpha = 100\,\%$: Even more jams on R2

Random TT contributions (σ_T =60 s) dampen oscillations



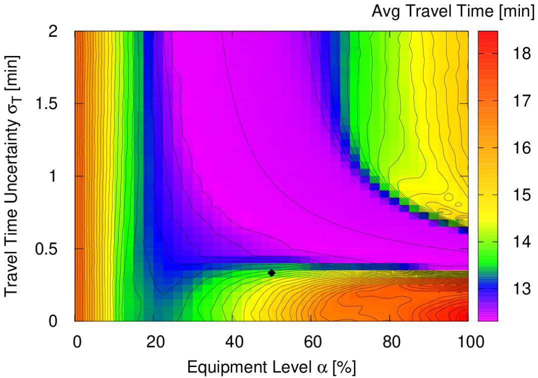
No equipped cars: Of course, the same

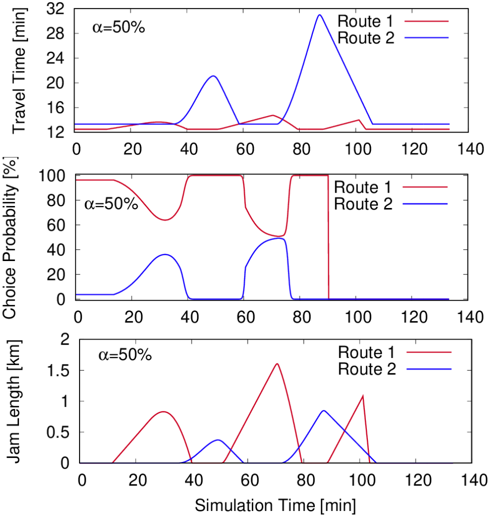
$\alpha = 20\,\%$: Assignment approaches the deterministic steady-state SO

$\alpha = 50\,\%$: Steady state right from the beginning. More vehicles to R2 than in deterministic UE/SO

$\alpha = 100\,\%$: Too high equipment levels lead to oscillations also with random times

Navigation with random times and delay





13.3 Microscopic Approach

Since the Logit model for the macroscopic choice probabilities reflects additive random time contributions, a microscopic simulation is simple:

- Same network
- A micromodel (e.g., the IDM) leading to the same travel times and capacities
- Instantaneous travel time: lane-average of the instantaneous lane travel time

$$T_l(t) = \sum_{i=1}^{n_l} \frac{\Delta x_i(t)}{v_i(t)}$$

- When crossing the decision point, take samples e_r of logistic distributed time uncertainties $\epsilon_1 - \epsilon_2$ at the decision point and take the route with the shortest time $T_r + e_r$
- Make use of the microscopic simulation to model decision thresholds, provider data uncertainties, different delays, ...

Microscopic simulation