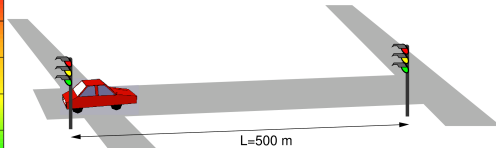
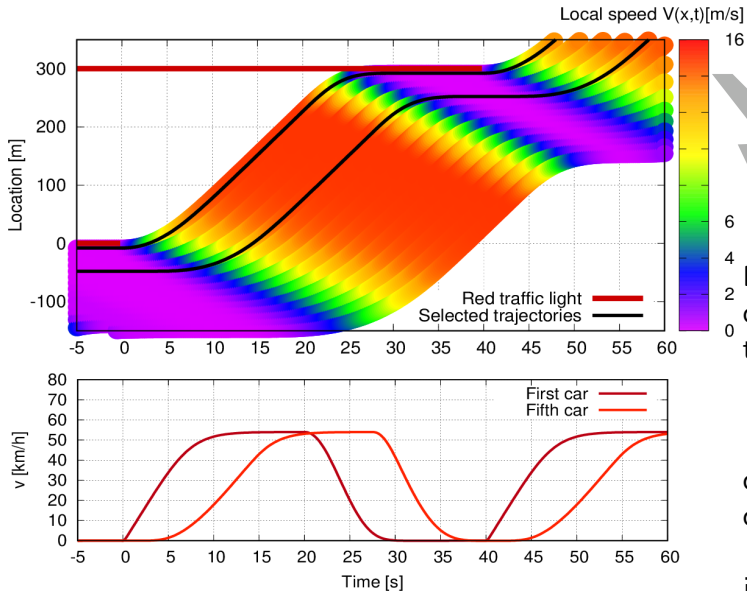


Lecture 08: Microscopic Models I

Elementary Car-Following Models

- ▶ 8.1 Difference between Micro and Macromodels
- ▶ 8.2 Types and Mathematical Forms
- ▶ 8.3 Car-Following Models
- ▶ 8.4 Optimal Velocity Model
- ▶ 8.5 Full Velocity Difference Model
- ▶ 8.6 Newell's Car-Following Model
- ▶ 8.7 Car-Following Cellular Automata

8.1 Difference between Micro and Macromodels



Microscopic:

describes the trajectories or FC time series

Macroscopic:

describes the inverse of the local distance of the lines (**density**) or the local gradient of the trajectories (**local speed**)

Characterisation of Microscopic Models

- ▶ Generally, **microscopic models** consider the smallest objects that make sense/play a role in the given context, e.g., molecules/atoms/elementary particles in physics or individual decision makers in economics.
- ▶ In traffic flow, this smallest object usually is the **driver-vehicle unit** (*why vehicle and driver?*) but it can also be a cyclist, a pedestrian, or others.
- ▶ Microscopic models are more detailed than the macroscopic models discussed in the previous sections which locally aggregate the microscopic quantities.
- ▶ Microscopic models are less detailed than models for the **vehicle dynamics** (“submicroscopic models”) treating aspects such as brake and engine control path, slip, or stability control

Where micromodels play out their advantages: heterogeneous traffic

Microscopic models play out their advantages when describing different **driver-vehicle units**, i.e., **heterogeneous traffic**. They are also called **self-driven particles** or **agents** (no stirring or shaking involved!).

There are four conceptual levels for heterogeneity that all can be tackled:

- ▶ *Same model, same vehicle category, same driving style*: Since drivers are no machines, some **acceleration noise** is plausible.
- ▶ *Same model, same vehicle category (e.g., only cars or only trucks), different driving styles (e.g. considerate or aggressive)*: every agent gets its individual parameter set drawn from a distribution
- ▶ *Same model, different vehicle categories, different styles*: The agents of each category get their parameters from separate distributions
- ▶ *Different models*: Fundamentally different agents such as human vs. autonomous driving, cycles, tuctucs/motor-rickshaws, cars/trucks

8.2 Microscopic Traffic Flow Models: Types and Mathematical Forms

- ▶ Generally, microscopic traffic flow models can describe any aspect of the dynamics of a driver and his/her vehicle on two levels:
 - ▶ **Operative level**: accelerating, braking, steering
 - ▶ **Tactical levels**: lane changing, entering a priority road and other discrete-choice tasks
 - ▶ **Strategic level**: route choice
- ▶ Hence, there are different model categories:
 - ▶ **Car-following (CF) models** or more generally models for the longitudinal dynamics are the most important representatives of microscopic traffic flow models
 - ▶ **lane-changing models** or **integrated models** (combining longitudinal and lateral dynamics)
 - ▶ **non-lane-based models**, e.g., for mixed traffic (India), cross-country skiing and running events,
 - ▶ general **discrete-choice models** for situations such as entering or crossing a road, stopping behind a traffic light
 - ▶ higher-level micromodels for whole routes: **multi-agent models**

Mathematical forms

- ▶ Continuous in space and time: **coupled ordinary differential equations (ODEs)** as in Newtonian dynamics:

$$\frac{dx_i}{dt} = v_i, \quad \frac{dv_i}{dt} = f_i(x_i, x_{i-1}, v_i, v_{i-1}, \dots)$$

Why $f_i(\cdot)$ instead of $f(\cdot)$? Different driving styles or even models

- ▶ Discrete update timesteps: **iterated maps**

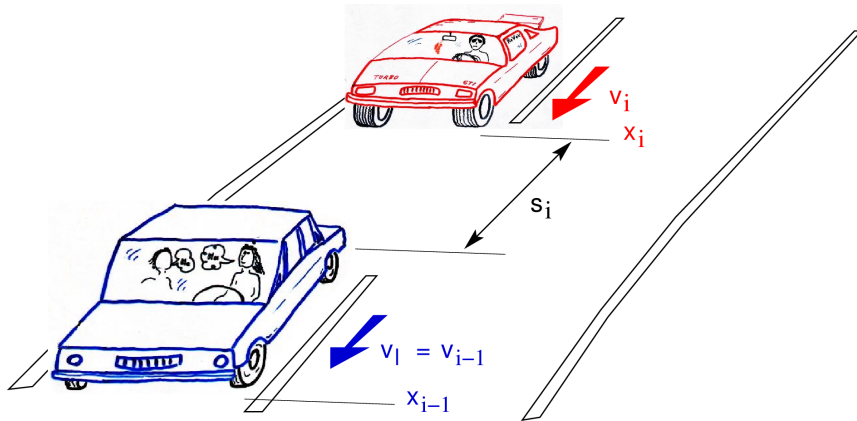
$$x_i(t + \Delta t) = f_i^x(x_i(t), v_i(t)), \quad v_i(t + \Delta t) = f_i^v(x_i(t), x_{i-1}(t), v_i(t), v_{i-1}(t), \dots)$$

- ▶ Space, time, and state are all discrete: **cellular automata (CA)**

$$\mathbf{v}(t + 1) = \mathbf{f}^{\text{CA}}(\mathbf{v}(t)), \quad v_k = \begin{cases} -1 & \text{cell } k \text{ empty} \\ 0, 1, \dots & \text{cell } k \text{ occupied,} \\ & \text{speed } v_k^{\text{phys}} = v_k \Delta x / \Delta t \end{cases}$$

- ? Give the frame of reference (Euler or Lagrange) of each mathematical form CA: Euler; the others: Lagrange

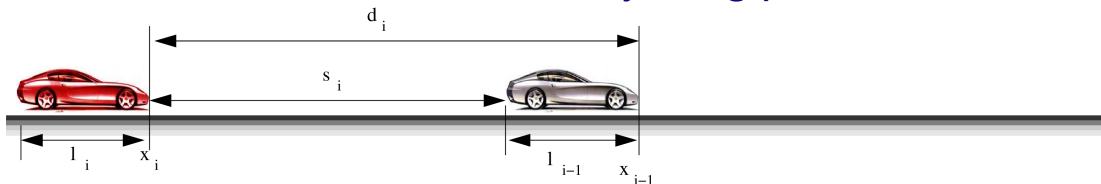
8.3 Car-Following Models



Most car-following models consider just the immediate leader, exactly like an **adaptive-cruise control (ACC)** system:

- ▶ Independent variables: speed v_i , gap $s_i = x_{i-1} - x_i - l_{i-1}$, and leading speed $v_{i-1} := v_l$
- ▶ Position x_i : front bumper of vehicle i , increasing in driving direction
- ▶ Indices i as in a race: the first becomes Number 1, so $x_{i-1} > x_i$

Clarification: headways and gaps



- ▶ **Headways** always denote differences including the vehicle's occupancy time or length:
 - ▶ The **time headway** or simple **headway** $\Delta t_i = t_i - t_{i-1}$ gives the time interval between consecutive vehicles passing a fixed spot
 - ▶ The **distance headway** $d_i = x_{i-1} - x_i$ gives the distance of the vehicle fronts between leader and follower at a fixed time

- ▶ **Gaps** always denote the bumper-to-bumper differences
 - ▶ The **time gap** $T_i = t_i - t_{i-1} - l_{i-1}/v_{i-1}$ gives the time interval of no occupation between leader and follower at a fixed spot. It is the time headway minus the leader's occupancy time
 - ▶ The **distance gap** or simply **gap** $s_i = x_{i-1} - x_i - l_{i-1}$ gives the bumper to bumper gap, i.e., distance headway minus the leader's vehicle length

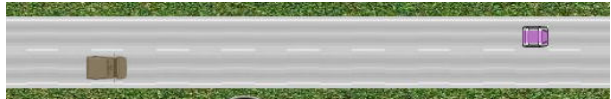
- ▶ The **time to collision** $T_i^c = s/(v_i - v_{i-1})$ gives exactly that if $v_i > v_{i-1}$ and there are no accelerations.

Model plausibility and completeness

A (generalized) car-following model is **complete** if it is able to realistically describe free flow and all common steady-state and dynamic situations with a leader

Free flow:

- ▶ realistic acceleration profile
- ▶ existence of a desired speed v_0



Steady-state:

- ▶ existence of a minimum gap
- ▶ following a leader at a plausible time gap
- ▶ transition to the free-flow state for sufficiently large gaps



Model plausibility and completeness II

dynamic situations:

- ▶ when closing in, regular transition to a car-following situation
- ▶ when approaching a stopped obstacle (vehicle queue or red traffic light), regular deceleration to a stop at some minimum gap
- ▶ handling of a target change (cutting in and out of leaders)
- ▶ handling of emergency situations (transition to closing in)



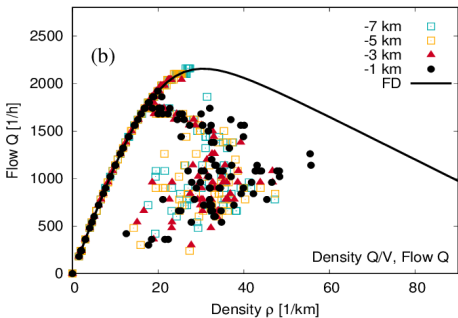
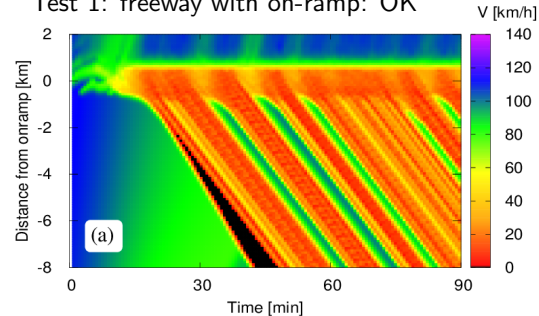
collective phenomena:

- ▶ traffic breakdown at situations where it is observed
- ▶ traffic flow instabilities
- ▶ formation of traffic waves with the right properties
- ▶ Producing the right flow-density data from *virtual stationary detectors*

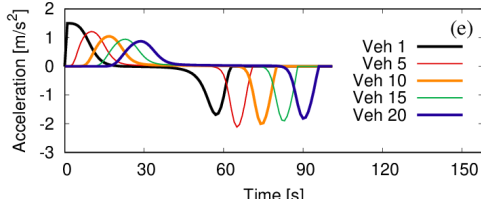
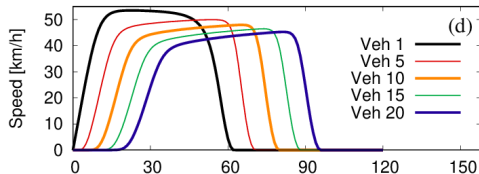
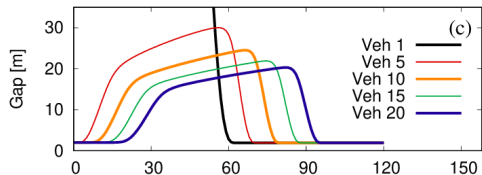


Example of a complete model: IDM

Test 1: freeway with on-ramp: OK

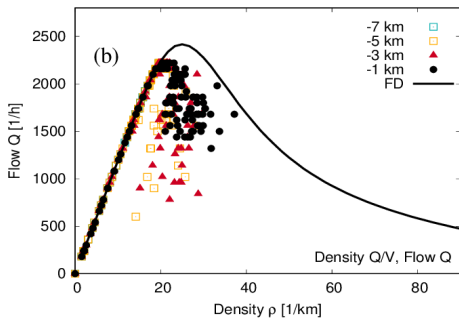
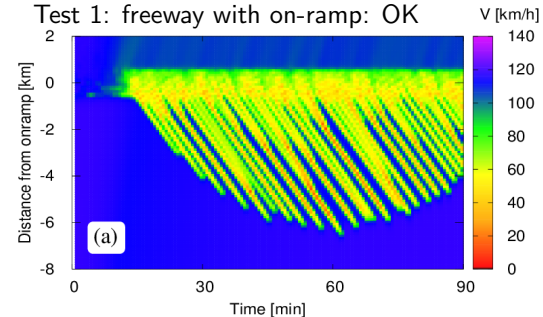


Test 2: traffic lights: OK

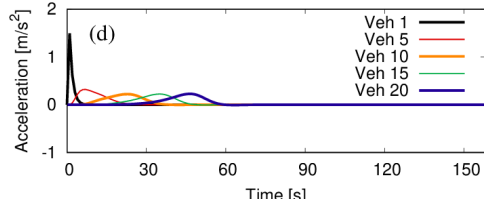
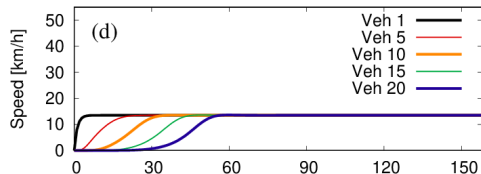
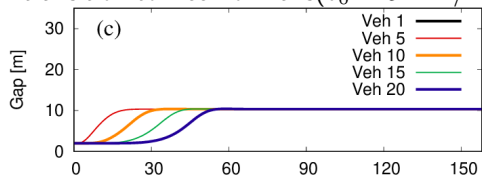


Example of an incomplete model: FVDM

Test 1: freeway with on-ramp: OK



Test 2: traffic lights:
transition to free flow fails ($v_0 = 54$ km/h)



Plausibility criteria: the acceleration function

Formulate both ODE and iterated map models such that $f(\cdot)$ stands for the acceleration function:

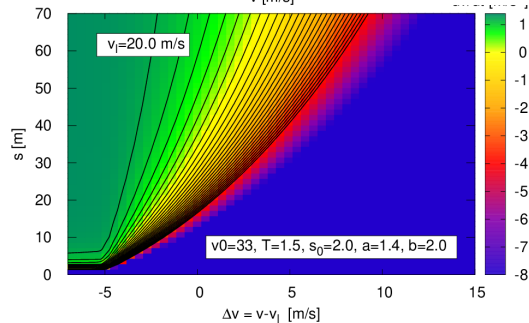
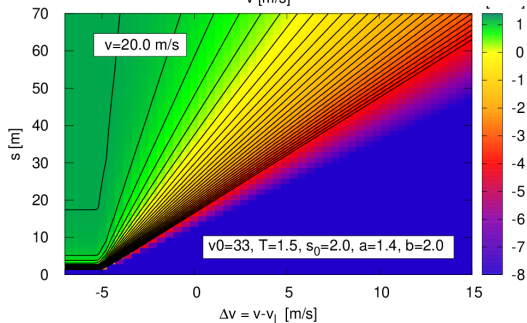
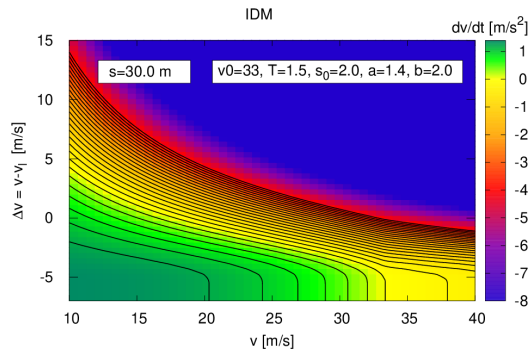
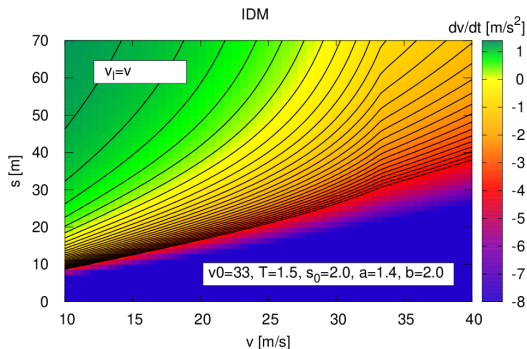
- ▶ ODE models:

$$\frac{dx_i}{dt} = v_i, \quad \frac{dv_i}{dt} = f(s_i, v_i, v_{i-1}) \equiv f(s, v, v_l)$$

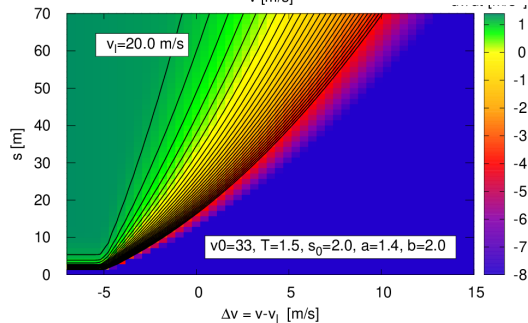
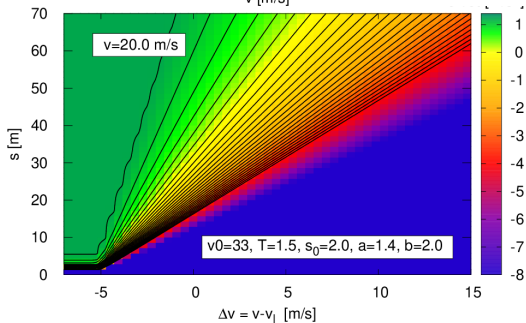
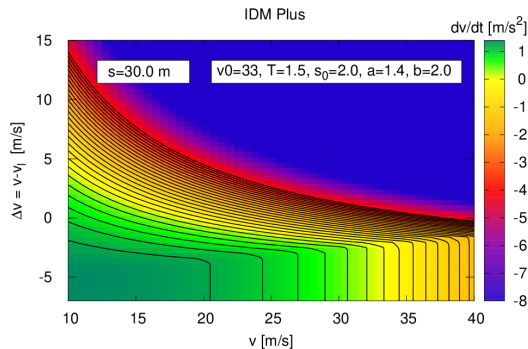
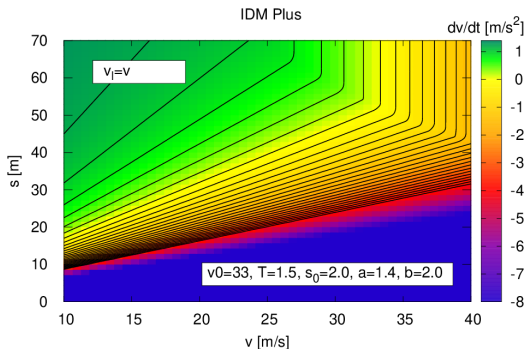
- ▶ Iterated-map models:

$$\begin{aligned} v_i(t + \Delta t) &= v_i(t) + f(s_i(t), v_i(t), v_{i-1}(t)) \Delta t, \\ x_i(t + \Delta t) &= x_i(t) + \frac{1}{2} [v_i(t) + v_i(t + \Delta t)] \Delta t \end{aligned}$$

Plausibility criteria: the IDM acceleration function



Plausibility criteria: the IDMplus acceleration function



Plausibility criteria I

A necessary condition for completeness is that the following **plausibility conditions** are satisfied:

- (1) Dependence of the acceleration on the own speed and existence of a desired speed v_0 :

$$\frac{\partial f(s, v, v_l)}{\partial v} < 0, \quad \lim_{s \rightarrow \infty} f(s, v_0, v_l) = 0$$

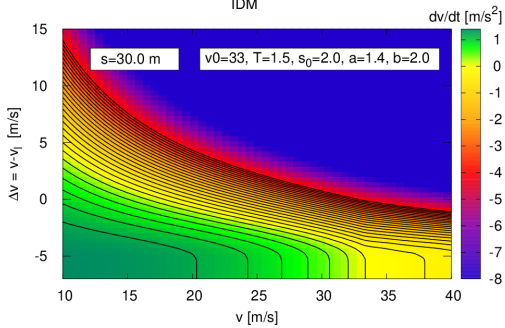
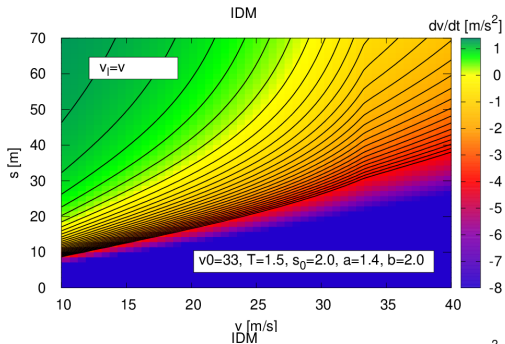
- (2) Dependence on the gap with limiting case of no interaction:

$$\frac{\partial f(s, v, v_l)}{\partial s} \geq 0, \quad \lim_{s \rightarrow \infty} \frac{\partial f(s, v, v_l)}{\partial s} = 0$$

- (3) Dependence on the leader's speed:

$$\frac{\partial f(s, v, v_l)}{\partial v_l} \geq 0, \quad \lim_{s \rightarrow \infty} \frac{\partial f(s, v, v_l)}{\partial v_l} = 0, \quad \left| \frac{\partial f}{\partial v_l} \right| \leq \left| \frac{\partial f}{\partial v} \right|$$

Plausibility criteria II: Steady-state relation



Steady-state speed-gap relation and existence of a minimum gap:

The steady-state speed $v_e(s)$ defined by $f(s, v_e(s), v_e(s)) = 0$ satisfies

$$v'_e(s) \geq 0, \quad \lim_{s \rightarrow \infty} v_e(s) = v_0, \quad v_e(s_0) = 0 \text{ for some } s_0 > 0$$

Express $v'_e(s)$ in terms of $\frac{\partial f}{\partial s}$, $\frac{\partial f}{\partial v}$, and $\frac{\partial f}{\partial v_l}$ and show that this condition follows from (1) and (2)

$$f(s_e, v, v) = 0$$

$$\begin{aligned} \Rightarrow 0 &= df \\ &= \frac{\partial f}{\partial s} ds + \frac{\partial f}{\partial v} dv + \frac{\partial f}{\partial v_l} dv \\ &= \left(\frac{\partial f}{\partial s} + \frac{\partial f}{\partial v} v'_e(s) + \frac{\partial f}{\partial v_l} v'_e(s) \right) ds \end{aligned}$$

$$\Rightarrow v'_e(s) = -\frac{\partial f}{\partial s} / \left(\frac{\partial f}{\partial v} + \frac{\partial f}{\partial v_l} \right)$$

≥ 0 since $\frac{\partial f}{\partial s} \geq 0$, $\frac{\partial f}{\partial v} < 0$, and $\left| \frac{\partial f}{\partial v_l} \right| \leq \left| \frac{\partial f}{\partial v} \right|$
and $v_e(s \rightarrow \infty) = v_0$ from (1)

Some Examples of Elementary Car-Following Models

- ▶ Not really useful for actually simulating traffic flow
- ▶ but very good for showing the basic principles,
- ▶ also serve as basis for the more sophisticated ones

8.4 Optimal Velocity Model

8.5 Full Velocity Difference Model

8.6 Newell's Car-Following Model

8.7 Car-Following Cellular Automata

8.4 Optimal Velocity Model (OVM)

$$\frac{dv}{dt} = \frac{v_{\text{opt}}(s) - v}{\tau} \quad \text{Optimal Velocity Model}$$

Whole model class parameterized by the **optimal-velocity function** $v_{\text{opt}}(s)$, e.g.,

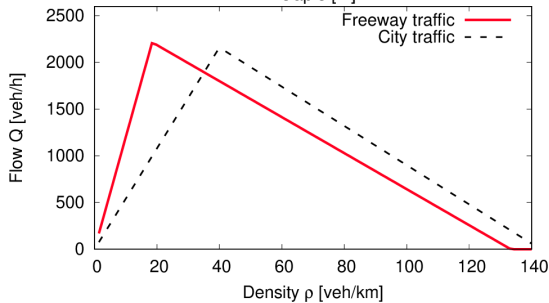
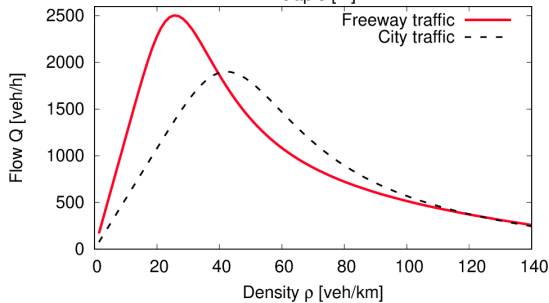
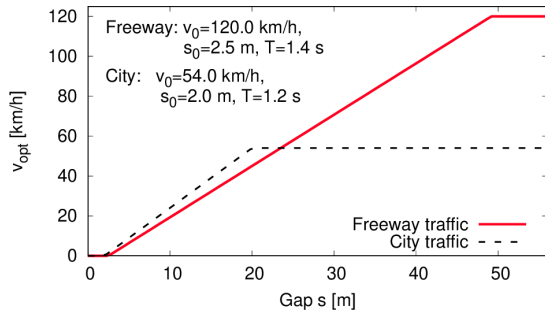
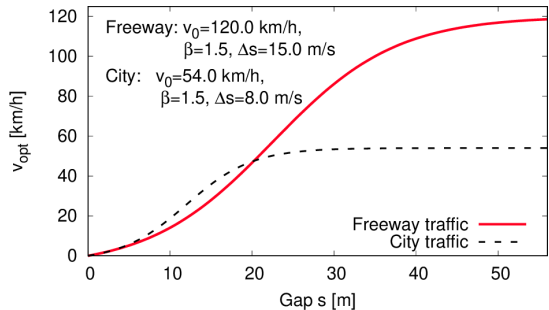
- ▶ Original OVM function by Bando et al:

$$v_{\text{opt}}(s) = v_0 \frac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh \beta}{1 + \tanh \beta}$$

- ▶ OVM function corresponding to the triangular FD:

$$v_{\text{opt}}(s) = \max \left[0, \min \left(v_0, \frac{s - s_0}{T} \right) \right]$$

OV functions



Bando OV function

triangular OV function

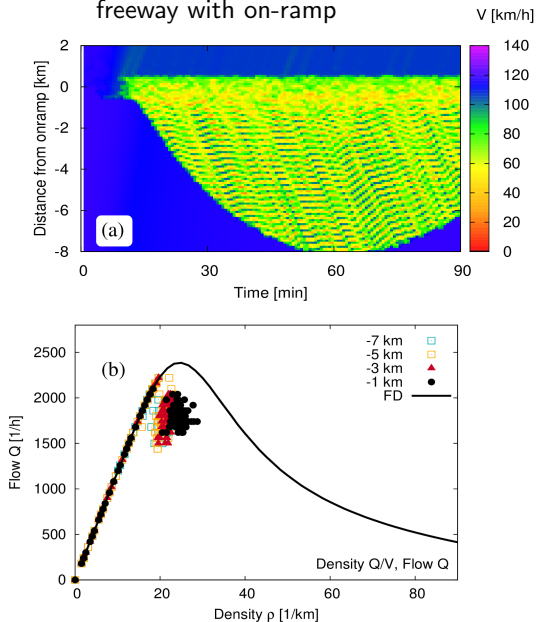
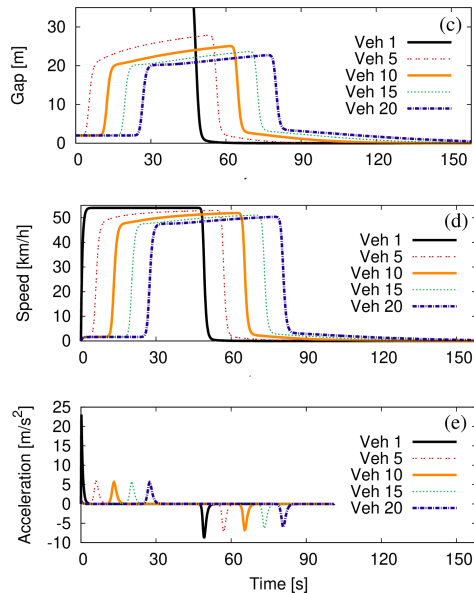
Properties of the Optimal Velocity Model (OVM)

- ▶ The homogeneous-steady-state speed $v_e(s)$ is given by the OV function
- ▶ Technically, the model *marginally* satisfies all plausibility conditions (no sensitivity to the leader's speed) but results in unrealistic accelerations, or crashes, or both
- ▶ Besides the parameters of the OV function, the OVM has the **speed relaxation time** τ as additional parameter:
 - ▷ The more responsive the driver, the lower τ ,
 - ▷ the higher τ , the more instabilities

Parameter	Typical Value Highway	Typical Value City Traffic
Adaptation time τ	0.65 s	0.65 s
Desired speed v_0	120 km/h	54 km/h
Transition width Δs (Bando FD)	15 m	8 m
Form factor β (Bando FD)	1.5	1.5
Time gap T (triangular FD)	1.4 s	1.2 s
Minimum distance gap s_0 (triangular FD)	3 m	2 m

Factsheet of the Optimal Velocity Model (OVM)

freeway with on-ramp


 city with traffic lights
 extreme accelerations!


OVM questions $f_{\text{OVM}}(s, v, v_l) = (v_{\text{opt}}(s) - v)/\tau$

OV functions: $v_{\text{opt}}^{\text{Bando}} = v_0 \frac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh \beta}{1 + \tanh \beta}$, $v_{\text{opt}}^{\text{triang}} = \max\left[0, \min\left(v_0, \frac{s - s_0}{T}\right)\right]$

? Show that the steady state speed $v_e(s)$ is given by the optimal speed.

! Steady State $v = v_l$, $\frac{dv}{dt} = 0$: $0 = (v_{\text{opt}}(s) - v)/\tau$. Since the speed adaptation time $\tau > 0$, we have $v = v_e(s) = v_{\text{opt}}(s)$

? Check the plausibility conditions

! (1) $\frac{df}{dv} = -1/\tau < 0$ OK

(2) $\frac{df}{ds} = v'_e(s)/\tau \geq 0$ if $v'_e(s) \geq 0$ OK

(3) $\frac{df}{dv_l} = 0$ marginally OK

(4a) Bando OV function: $v'_{\text{opt}}(s) \geq 0$ since $\tanh(\cdot) \geq 0$, $v_{\text{opt}}(s \rightarrow \infty) = v_0$, $v_{\text{opt}}(0) = 0$ (OK)

(4b) triangular OV function: $v'_{\text{opt}}(s) = 1/T$ or $=0$, $v_{\text{opt}}(s \rightarrow \infty) = v_0$, $v_{\text{opt}}(s_0) = 0$ OK

? show that the “triangular” OV function in fact leads to the triangular FD

! triangular FD: $Q(\rho) = \rho v_{\text{opt}}(1/\rho - l - s_0) = \rho \max[0, \min(v_0, (1/\rho - l)/T)] = \max[0, \min(v_0\rho, 1/T(1 - \rho l))]$
 $= \max[0, \min(v_0\rho, 1/T(1 - \rho/\rho_{\text{max}}))]$

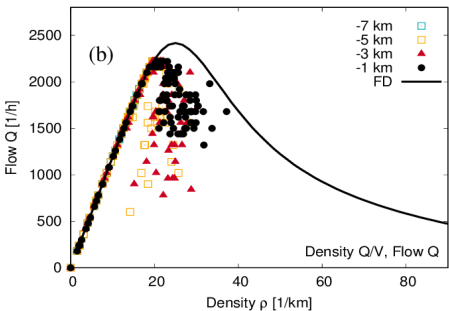
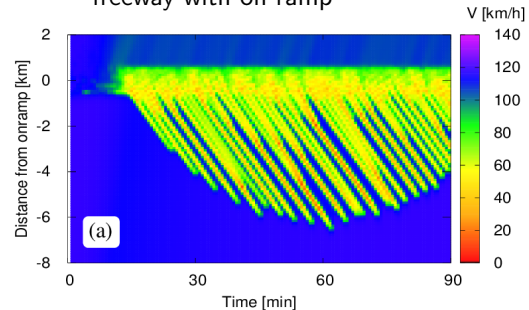
8.5. Full Velocity Difference Model (FVDM)

$$\frac{dv}{dt} = \frac{v_{\text{opt}}(s) - v}{\tau} + \gamma(v_l - v) \quad \text{Full Velocity Difference Model}$$

- ▶ The FVDM is the Optimal Velocity Model with an additional sensitivity to the relative speed $v - v_l$ to the leader
- ▶ The additional sensitivity parameter γ has values of the order of 0.5 s^{-1}
- ▶ As in the OVM, the homogeneous steady state speed $v_e(s) = v_{\text{opt}}(s)$
- ▶ As a pure car-following model, the FVDM behaves more realistically. However, in contrast to the OVM, it is *not* complete **Why?** For $s \rightarrow \infty$, the FVDM acceleration still depends strongly on v_l thereby violating plausibility requirement (3b) $\lim_{s \rightarrow \infty} \frac{\partial f}{\partial v_l} = 0$: There is no transition from car-following to free traffic

Factsheet of Bando's Full Velocity Difference Model (FVDM)

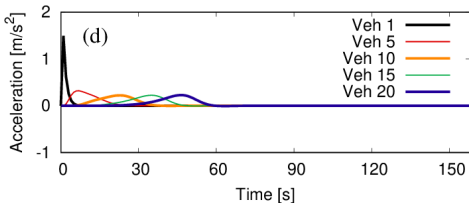
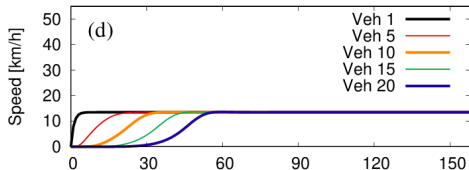
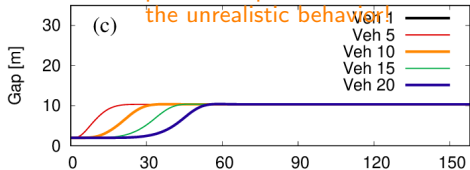
freeway with on-ramp



city with traffic lights

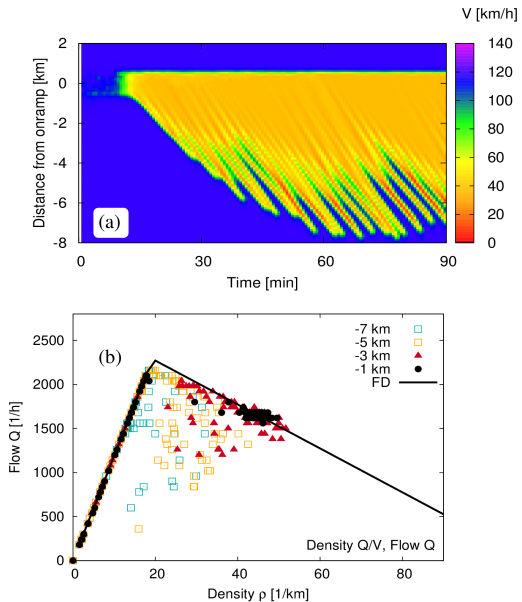
spot and explain

the unrealistic behavior!

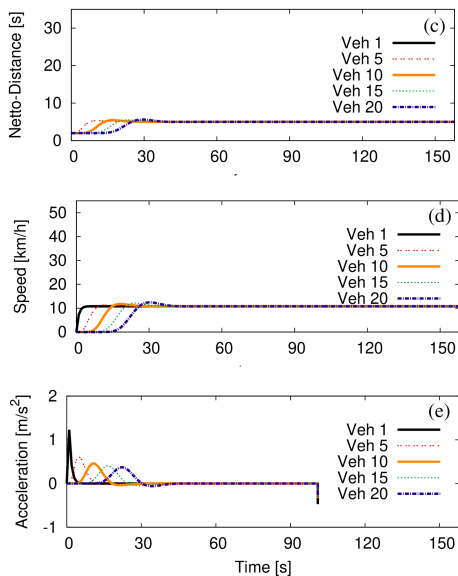


Factsheet of the FVDM with triangular FD

freeway with on-ramp

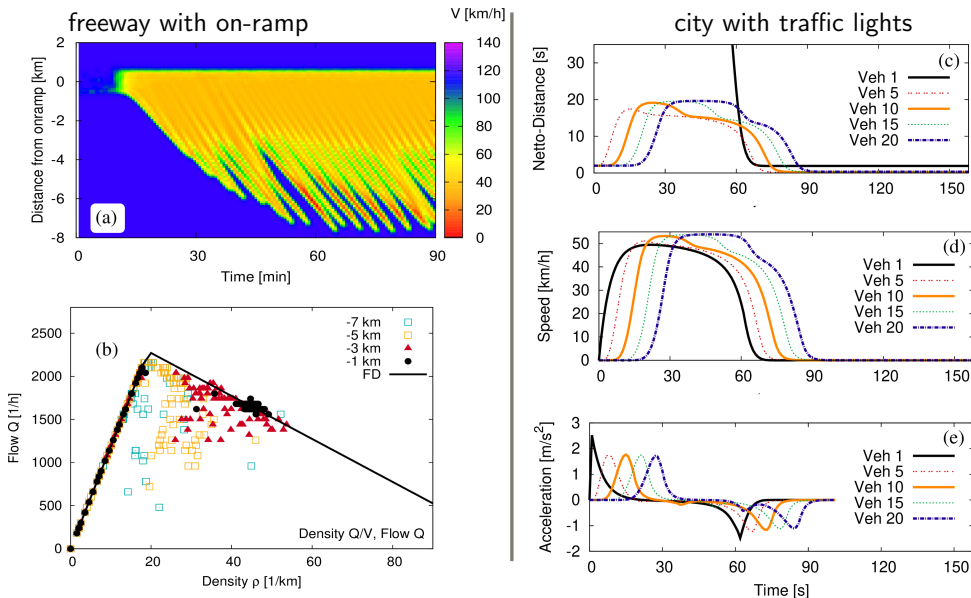


city with traffic lights



Factsheet of the **modified FVDM** with triangular FD

$$f(s, v, v_l) = (v_{\text{opt}}^{\text{triang}} - v)/\tau + \gamma(v_l - v) \min(1, v_0 T/s)$$



8.6 Newell's Car-Following Model

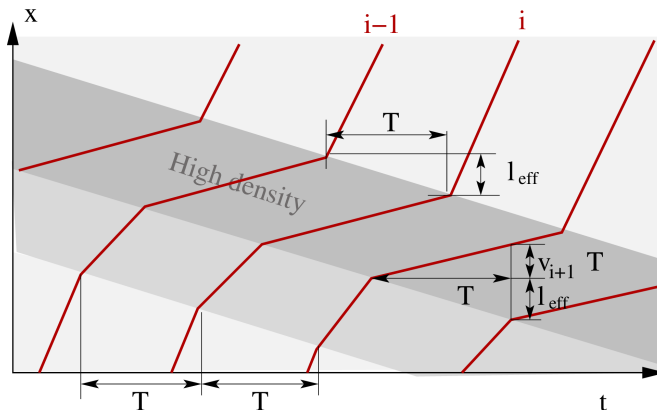
$$v(t + T) = v_{\text{opt}}(s(t)), \quad v_{\text{opt}}(s) = \min\left(v_0, \frac{s}{T}\right) \quad \text{Newell's Model}$$

- ▶ The OV relation can also be written in terms of the distance headway $\tilde{v}_{\text{opt}}(d) = v_{\text{opt}}(s + l_{\text{eff}})$ and represents the triangular FD (**check!**)

$$Q(\rho) = \min\left[V_0\rho, \frac{1}{T}(1 - \rho l_{\text{eff}})\right]$$

- ▶ Three parameters: effective vehicle length l_{eff} (incl minimum gap s_0), reaction time T , and desired speed v_0
- ▶ T is not only the reaction time but also the time gap, the speed adaptation time, and the numerical update timestep (**check!**)

Newell's car-following model: properties



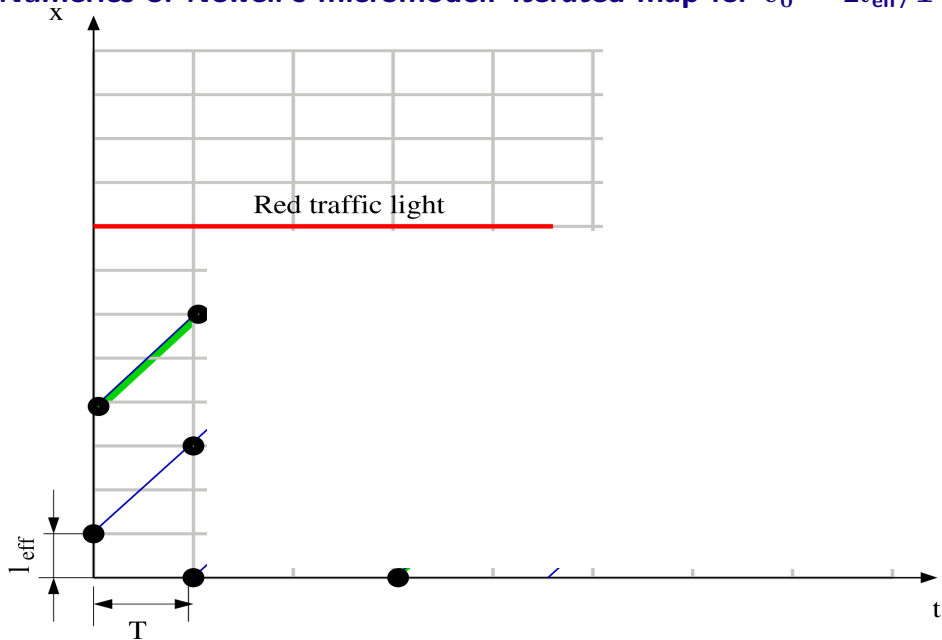
- ▶ Constant **wave speed** w by considering the start of a queue of standing vehicles (distance headway $d = l_{\text{eff}}$) or simply by the general expression $w = Q'_{\text{cong}}(\rho)$ from the congested part of the FD:

$$w = -l_{\text{eff}}/T$$

- ▶ This means that, in the car-following regime ($s/T < v_0$), the follower adopts the leader's speed one "reaction time" T ago and proceeds by the gap value one "reaction time" T ago:

$$v(t+T) = v_l(t), \quad x(t+T) = x_l(t) - l_{\text{eff}}$$

Numerics of Newell's micromodel: iterated map for $v_0 = 2l_{\text{eff}}/T$



8.7. Car-Following Cellular Automata (CA)

Cellular automata (CA) describe all aspects of dynamical systems by using (generally small) integers:

- ▶ Space is subdivided into cells
- ▶ Time is subdivided into time steps Δt
- ▶ State variables are multiples of the natural unit, e.g., speed in cells/ Δt and accelerations in cells/ $(\Delta t)^2$
- ▶ In the **Euler** or occupation number representation the dynamical unit is a cell that can be occupied (1) or not (0) [here, the maximum speed $v_0 = 1$ and we have redefined the state $-1 \rightarrow 0$ for empty, and 0 or $1 \rightarrow 1$ for occupied with speed 0 or 1 to match the historic example] such as in the famous Rule 184 ($= 2^7 + 2^5 + 2^4 + 2^3$) (try to understand it):

current local pattern	7=	6=	5=	4=	3=	2=	1=	0=
	111	110	101	100	011	010	001	000
new state of the center cell	1	0	1	1	1	0	0	0

- ▶ In the **Lagrange** representation a CA looks like a discretized car-following model such as the **Nagel-Schreckenberg Model** below

Nagel-Schreckenberg Model (NSM) and the Barlovic Model

These are **Stochastic** CAs in the Lagrange representation, i.e., the relevant unit is a vehicle i rather than a cell k :

1. *Deterministic acceleration* as a function of the speed v_i , desired speed v_0 and gap (number of empty cells) g_i :

$$v_i^*(t+1) = \min(v_i(t) + 1, v_0, g_i)$$

2. *Dawdling* by not accelerating, or braking more than necessary, with a certain dawdling probability p :

$$v_i(t+1) = \begin{cases} \max(v_i^*(t+1) - 1, 0) & \text{with probability } p, \\ v_i^*(t+1) & \text{otherwise.} \end{cases}$$

In the Barlovic model, the “slow-to-start” rule applies, i.e., the probability p_0 for standing vehicles ($v_i(t) = 0$) is higher than p for driving vehicles

- ▶ *Driving* by moving $v_i(t+1)$ cells forward:

$$x_i(t+1) = x_i(t) + v_i(t+1).$$

Verify that *Rule 184* corresponds to the deterministic NSM with $v_0 = 1$

Then, a car moves by one cell whenever the new cell is free. Compare with the Rule-184 table

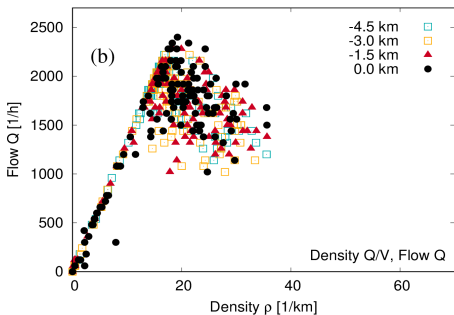
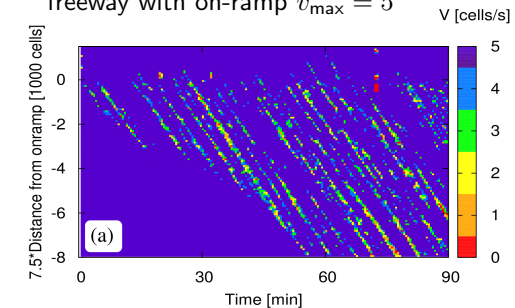
How the NSM works ($v_0 = 2$)



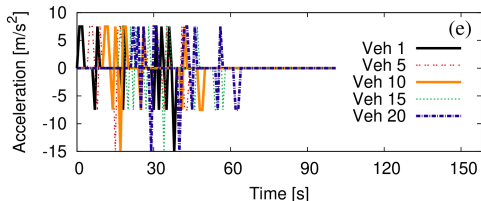
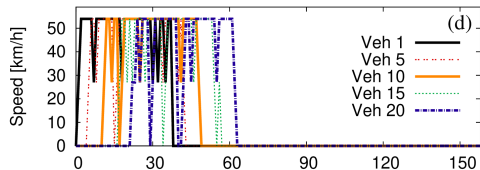
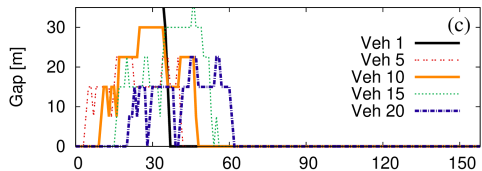
Parameter	Typ.	Value	Typ.	Value
	Highway		City	
Cell length $\Delta x_{\text{phys}} = l_{\text{eff}}$		7.5 m		7.5 m
Time step Δt_{phys}		1 s		1 s
Desired speed v_0		5		2
Dawdling probability p		0.2		0.1
Prob. p_0 when stopped (Barlovic)		0.4		0.2

Factsheet of the Nagel-Schreckenberg Model (NSM)

freeway with on-ramp $v_{\max} = 5$

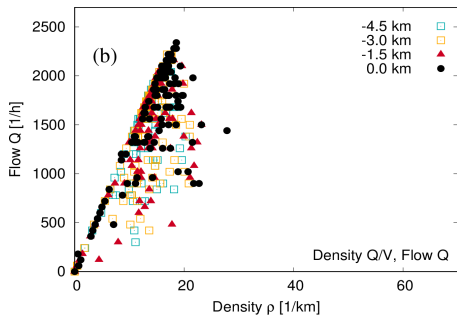
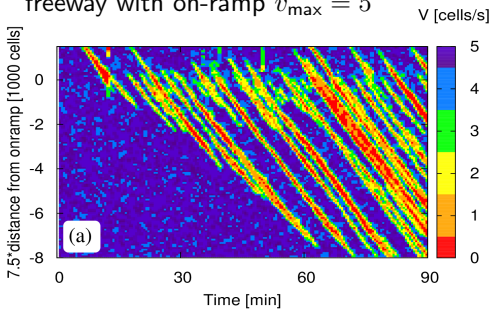


city with traffic lights $v_{\max} = 2$

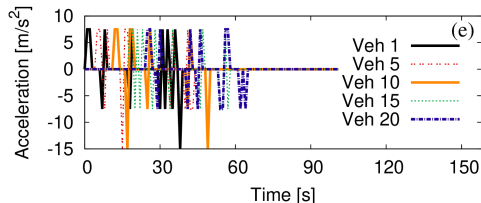
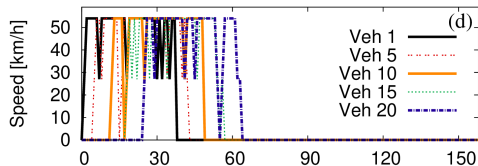
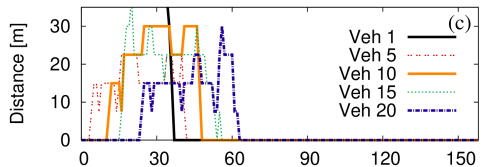


Factsheet of the CA model of Barlovic

freeway with on-ramp $v_{\max} = 5$



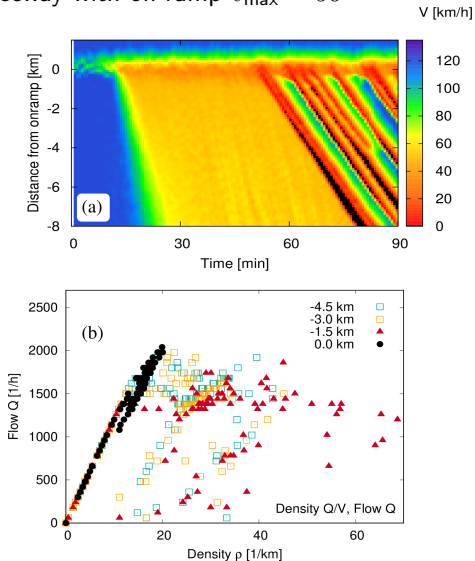
city with traffic lights $v_{\max} = 2$



Factsheet of the CA model of Kerner

There are many more “refined” CAs, e.g., the KCA with a cell size of only 0.5 m

freeway with on-ramp $v_{\max} = 56$



city with traffic lights $v_{\max} = 28$

