Traffic Flow Dynamics and Simulation

SS 2024, Solutions to Work Sheet 3, page 1

Solution to Problem 3.1: Locating a temporary bottleneck

(a) The helicopter flies against the driving direction at about 200 km/h. Its camera covers a road section of about 500 m.

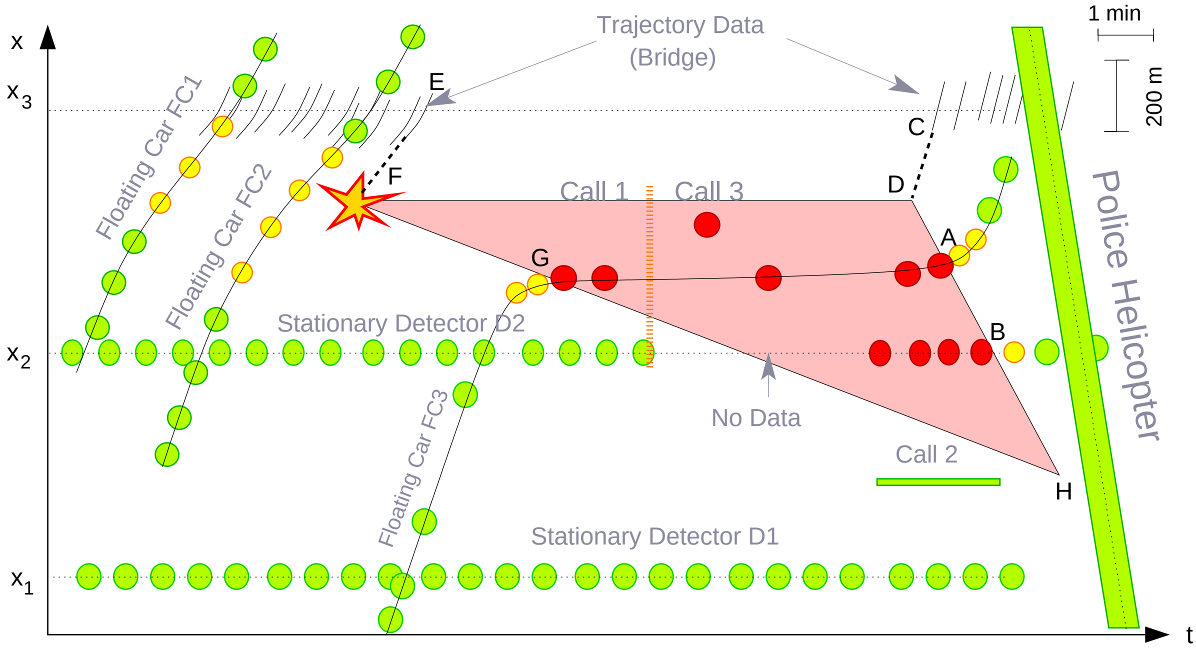
(b) Nowadays, information from private "traffic informers" is very up-to-date and gets more and more automatized thanks to "crowd-intelligence" applications as WAZE

- Caller 1 calls shortly, location uncertain, reports a jam
- Caller 2 is stationary (e.g., on a bridge), knows his/her position, reports free traffic the whole time
- Caller 3 reports a jam. It's a short call and the position is known

(c) From the data of floating car 3, we know that this car leaves a jam, i.e., crosses its downstream boundary, at the spatiotemporal point A depicted in the diagram below. From the data of detector 2 (point B), we know that this front is moving. We can exclude that the transition from congested to free traffic recorded by detector 2 at point B corrresponds to a downstream moving upstream front because (i) detector D1 records essentially constant traffic flow, (ii) the data of the detector D2 and the floating car 3 imply an upstream propagating upstream jam front, i.e., a growing jam. Hence, the upstream front is propagating backwards as long as it exists.

From the mentined stylized fact, we know that downstream fronts are either stationary or move at a constant velocity c_{cong}. Hence, the set of possible spatiotemporal points indicating when and where the road closure is lifted, lies on a line connecting the points A and B at a position $x > x_{\text{A}}$. The end of the road block not only sets the downstream jam front into motion but also leads to a transition empty road $\rightarrow$ maximum-flow state. The shockwave formula (**??**) implies that this front propagates with the desired free-flow speed v_0. Microscopically, this transition is given by the first car passing the accident site. This car is recorded as first trajectory of the trajectory data from the bridge at point C. Assuming, for simplicity, an instantaneous acceleration to the speed v_0, another set of possible spatiotemporal points for the removal of the road block is given by a line parallel to the first trajectory and touching it at point C (dashed line in the diagram). Intersecting the lines $\overline{\text{AB}}$ and the line parallel to the trajectories and going through C gives us the location and time of the lifting of the road block by the intersecting set of the two lines (point D), and also the location of the accident.

To estimate the time when the accident occurred, we determine the intersection F of the line $x = x_D$ of the temporary bottleneck, and the line representing the extrapolation of the last trajectory (point E) to locations further upstream (dashed line). Finally, because of the constant inflow recorded by D1, we know from the shockwave formula that the upstream jam front propagates essentially at a constant velocity, i.e., it is given by the line intersecting F and G. The jam dissolves when the upstream and downstream fronts meet at point H.



Solution to Problem 3.2: Dealing with inconsistent information

(a) Using equal weights, $V = \frac{1}{2}(V_1 + V_2)$, the error variance is

$$\sigma_V^2 = \frac{1}{4}\left(\sigma_1^2 + \sigma_2^2\right) = \frac{1}{4}\left(\sigma_1^2 + 4\sigma_1^2\right) = \frac{5}{4}\sigma_1^2\,,$$

assuming negligible systematic errors and independent random errors. Consequently, the error increases by a factor of $\sqrt{5/4}$ due to the inclusion of the noisy floating-car data. Using optimal weights,

$$V_{\text{opt}} = \frac{1}{5}(4V_1 + V_2)\,,$$

yields the error variance

$$(\sigma_V^2)_{\text{opt}} = \frac{1}{25}\left(16\sigma_1^2 + \sigma_2^2\right) = \frac{1}{25}\left(16\sigma_1^2 + 4\sigma_1^2\right) = \frac{4}{5}\sigma_1^2.$$

This means, adding floating-car data with a small weight to the stationary detector data *reduces* the uncertainty by a factor of down to $\sqrt{4/5}$, in spite of the fourfold variance of the floating-car data compared to the stationary detector data.

(b) This immediately leads to following *constrained optimization problem*: Minimize

$$\theta(\vec{r}) = \sum_m r_m^2 \theta_m, \tag{1}$$

subject to

$$\sum_m r_m = 1\,. \tag{2}$$

Constrained optimization problems can be solved using *Lagrange multipliers*. The procedure is as follows:

(i) Formulate each constraint n as a constraint function equating to zero, $B_n(\vec{r}) = 0$. Here, the only constraint $\sum_m r_m = 1$ results in the function $B_1(\vec{r}) = \sum_m r_m - 1$.

(ii) Define the *Lagrange function* by adding to the objective function to be minimized the constraint functions multiplied by Lagrange multipliers λ_n:

$$L(\vec{r}, \vec{\lambda}) = \theta(\vec{r}) - \sum_n \lambda_n B_n(\vec{r}), \tag{3}$$

where the vector $\vec{\lambda}$ represents the Lagrange multipliers which are unknown at this stage. In our optimization problem, the Lagrange function is given by $L(\vec{r}, \lambda_1) = \sum_m r_m^2 \theta_m - \lambda_1(\sum_m r_m - 1)$.

(iii) Find a necessary condition for the minimum of the Lagrange function with respect to $\vec{r}$:

$$\frac{\partial L(\vec{r}, \vec{\lambda})}{\partial \vec{r}} = 0. \tag{4}$$

This results in M equations if the weight vector $\vec{r}$ consists of M components. In our application, we obtain

$$\frac{\partial L}{\partial r_m} = 2 r_m \theta_m - \lambda_1 \stackrel{!}{=} 0 \quad \Rightarrow \quad r_m = \frac{\lambda_1}{2\theta_m}\,.$$

(iv) Determine the unknown Lagrange multipliers by applying the constraints. If there are N constraints, Eq. (4) and the constraints constitute $M + N$ equations for the $M + N$ unknown components of the vectors $\vec{r}$ and $\vec{\lambda}$. In our optimization problem, we obtain $\lambda_1 = 2/(\sum\limits_{m'} \theta_{m'}^{-1})$ resulting in the final weighting

$$r_m = \frac{\frac{1}{\theta_m}}{\sum\limits_{m'} \frac{1}{\theta_{m'}}}\,. \tag{5}$$

The weights should be proportional to the inverse of the variance of the errors in the data source.