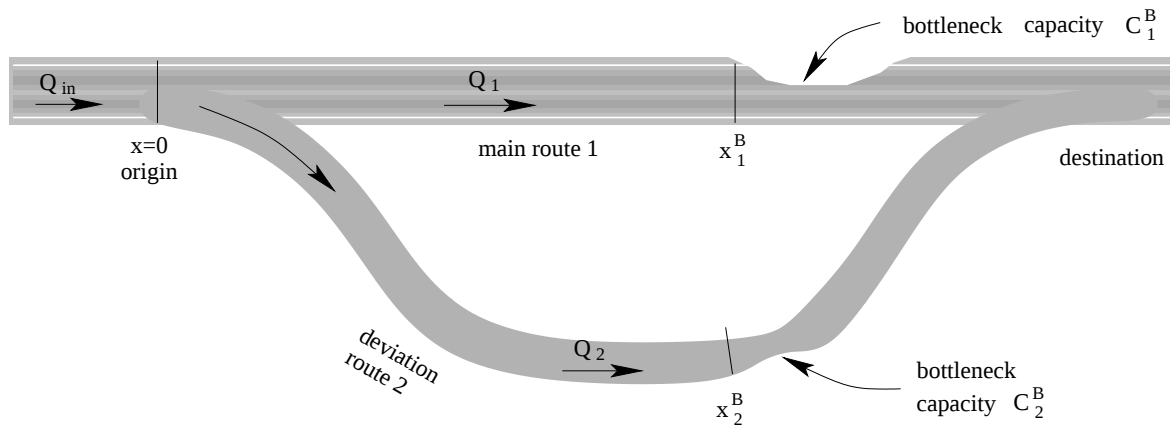


Traffic Flow Dynamics and Simulation

Summer semester, Tutorial 14, page 1

Problem 14.1: Dynamic Navigation

Given is a main route R1 (length from diverge to merge 15 km, 3 lanes, free speed $V_0 = 72$ km/h, LWR wave velocity $w = -18$ km/h, maximum density $\rho_{\max} = 150$ veh/km per lane) and a deviation route R2 of length 16 km from diverge to merge, one lane, and the same values for V_0 , w , and $\rho_{\max}$:



Furthermore, R1 and R2 have bottlenecks of capacities $C_1^B = 4860$ veh/h and $C_2^B = 1080$ veh/h, respectively that are located 14 km downstream of the merge (the precise position does not play a role, here). For simplicity, we assume no speed restrictions inside the bottlenecks. Use the LWR with a triangular fundamental diagram for the following tasks.

- Determine the travel times T_{10} and T_{20} for the empty network.
- Determine the user equilibrium (UE) $T_1(x) = T_{20}$ by calculating the length x of the jam on R1.
- Argue that the system optimum (SO), $Q_1^{\text{tot}} T_1 + Q_2^{\text{tot}} T_2 \stackrel{!}{=} \min$ leads to the same flow $Q_1^{\text{tot}} = C_1^B$ on R1 but, this time, there is no traffic jam. Also argue that this system optimum is unstable and will revert to the user equilibrium for perfectly informed drivers.
- Keeping aside oscillations, derive the following necessary conditions for the percentage α of drivers with active navigation devices (the drivers will always follow the recommendations) and travel-time uncertainties σ_T between the devices for realizing an UE or SO,

$$\text{UE: } (\alpha > P_2^{\text{SO}}) \text{ AND } \sigma_T \ll T_{02} - T_{01} \quad (1)$$

$$\text{SO: } (P_2 < 50\%) \text{ AND } \left(\alpha = \alpha_{\text{SO}} = P_2^{\text{SO}} \left(e^{\frac{T_{02} - T_{01}}{\sigma_T}} + 1 \right) \right) \quad (2)$$

where $P_2^{\text{SO}} = (Q_{\text{in}} - C_1^{\text{B}})/Q_{\text{in}} = 10\%$ is the assignment for SO (and also UE), $T_{02} - T_{01} = 50\text{ s}$ is the time difference for the empty network (and also in the SO). The time uncertainties are modelled with the *Logit* ansatz. For given *ground truth* instantaneous travel times T_1 and T_2 , the navigation devices recommend Route R2 with the probability

$$P_2^{\text{nav}} = \frac{1}{1 + \exp\left(\frac{T_1 - T_2}{\sigma_T}\right)},$$

so the percentage of deviating drivers is given by αP_2^{nav}

(e) Finally, derive following necessary condition for the Route 2 to remain uncongested,

$$\alpha \leq \alpha_{\text{max}} = P_2^{\text{max}} \left(e^{\frac{T_{02} - T_{01}}{\sigma_T}} + 1 \right), \quad (3)$$

with $P_2^{\text{max}} = C_2^{\text{B}}/Q_{\text{in}} = 20\%$, and show that, provided no oscillations build up, no jams on either route are observed if

$$\alpha_{\text{SO}}(\sigma_T) \leq \alpha \leq \alpha_{\text{max}}(\sigma_T). \quad (4)$$