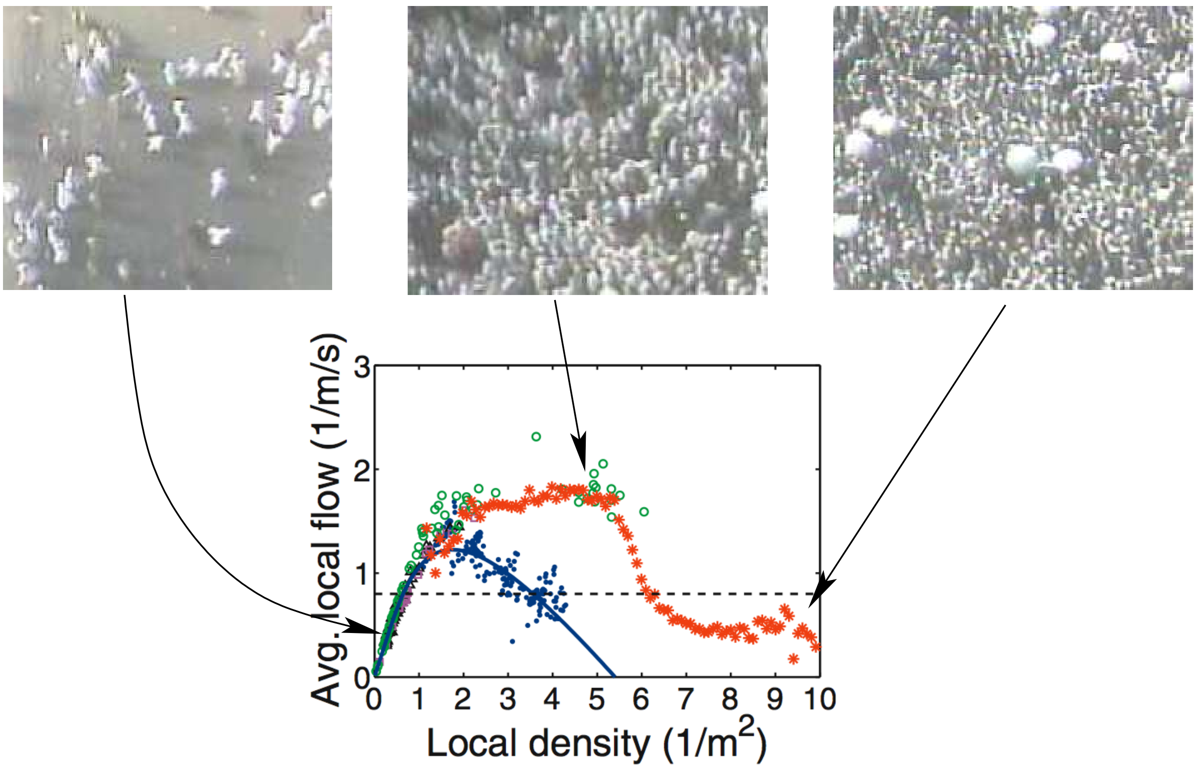
Traffic Flow Dynamics and Simulation

SS 2024, Tutorial 11, page 1

Problem 11.1: Pilgrimage in Mekka

Large unidirectional pedestrian flow such as in mass events are accessible to macroscopic modelling. Also flow-density scatter plots can be extracted such as that at the yearly Hajj in Mekka on the Jamarath Bridge (image and red symbols) or on other events in Europe (blue symbols).



(a) Discuss the units of flow Q, 2d density ρ, and flow density J

(b) In this case, pedestrian data have been derived from trajectories extracted from images such as that above. Do you expect systematic errors? If so, for which macroscopic quantities or which situations?

(c) Give possible prescriptions of how to measure these quantities from trajectories and arrive at scatter plots such as that in the figure

(d) Fit the parameters of (i) the triangular and (ii) the parabolic (Greenshields) fundamental diagrams $J(\rho)$ to the data of the European event (blue symbols) and the Hajj pilgrimage (red symbols).

(e) The images imply that, in congested traffic, the pedestrian crowd is rather isotropic, i.e., the lateral distances/gaps are about the same as the longitudinal ones, $\Delta x = \Delta y$ such that following relations hold for the 1d density ρ^{1d} and flow of a passage of width W:

$$\rho = (\rho^{\text{1d}})^2, \quad \Delta x = \Delta y = 1/\rho_{\text{1d}}, \quad Q = WJ((\rho^{\text{1d}})^2$$

It is tempting to use this relation for $W = \Delta y = 1/\rho_{\text{1d}}$ to describe single-file pedestrians. Argue why this is not a good idea by assuming that only the front, and not the side pedestrians give a repulsive interaction slowing the speed. Think of the pedestrian arrangement as a rectangular grid of single files at a space headway Δx from each other, and a distance Δy separated from the other single files and derive a plausible 1d fundamental diagram for single files. Determine the values for the triangular FD applied to the European event and also calculate the desired time gap of single files in this setting

Problem 11.2: Social Force Model (SFM)

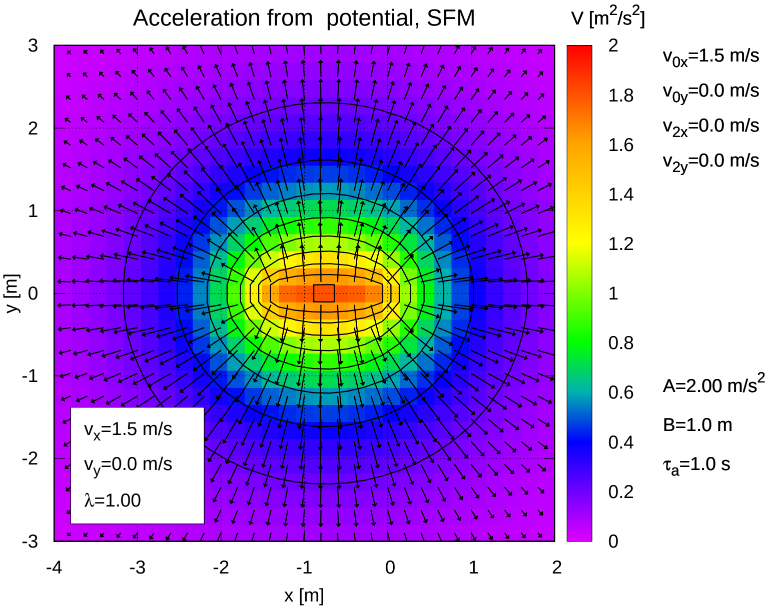
The walking dynamics of a pedestrian $i = 1$ as a response of the interaction with another pedestrian $j = 2$ shall be modelled with the interaction potential of the Social-Force Model (in the **elliptical specification II**)

$$\frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = \frac{\vec{v}_{0i} - \vec{v}_i}{\tau} - w(\phi_{ij}) \nabla_{\vec{x}_i} \Phi^{\text{int}}(\vec{x}_i) + f_{\text{boundaries}}$$

with the directionality

$$w(\phi) = \lambda + (1 - \lambda) \left(\frac{1 + \cos\phi}{2} \right)$$

($\phi = 0$ if the other pedestrian is ahead and $= \pi$ if in the back) and we neglect boundaries.



Interaction potential Φ^{int} (contour lines) of, and gradient forces (arrows) exerted by a pedestrian j standing at the origin $\vec{x}_j = (0, 0)$ to a pedestrian i at any position $\vec{x}$ walking at his/her desired velocity $\vec{v}_i = (v_0, 0)$

The interaction potential decreases exponentially with the semi-minor axis b of an ellipse (cf figure) with

- focal point 1 at the actual position $\vec{x}_j$ of pedestrian j,
- focal point 2 shifted by the anticipated distance change (as the distance, it is measured from i to j) $\Delta\vec{d} = (\vec{v}_i - \vec{v}_j)\Delta t$, assuming an anticipation time Δt and constant velocities,
- containing the position $\vec{x}_i$ of the subject pedestrian

If the other pedestrian is presently at the origin, $\vec{x}_j = \vec{0}$, the interaction potential reads

$$\Phi^{\text{int}}(x) = AB \ \exp\left(\frac{-b(\vec{x})}{B} \right)$$

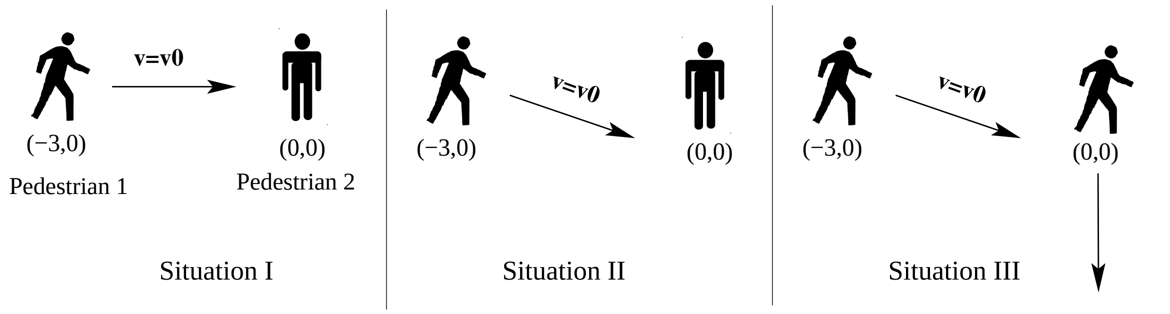
with

$$b(\vec{x}) = \frac{1}{2}\sqrt{\left(|\vec{x}| + |\vec{x} + \Delta\vec{d}|\right)^2 - |\Delta\vec{d}|^2}, \quad \Delta\vec{d} = (\vec{v}_i - \vec{v}_j)\Delta t$$

The SFM is parameterized to

$$B = 1\,\mathrm{m}, \quad A = 2\,\mathrm{m/s}^2, \quad \Delta t = 1\,\mathrm{s}, \quad \lambda = 0.06, \quad \tau = 2\,\mathrm{s}.$$

and shall be discussed in following three situations depicted in the figure:



In all situations, the initial speed of the considered pedestrian 1 is equal to his/her desired speed $v_0 = 5.4\,\mathrm{km/h}$. In scenario, he/she walks in the positive x direction, in the other scenarios, at an angle of 20 degrees in negative y direction. In all scenarios, pedestrian 2 is initially at the origin $\vec{x} = (0,0)$. In the first two scenarios, he/she stands while in Scenario 3, he walks at a speed of $3.6\,\mathrm{km/h}$ in negative y direction

(a) Discuss the parameters. Are the values plausible?

(b) Consider now Situation I where the subject pedestrian walks straight to a standing pedestrian or static obstacle. Simplify above equation for the interaction potential for this case for general values of $\vec{x}_1 = (-r, 0)$ and $\vec{v}_1 = (v, 0)$ assuming $r > 0$ ($x_1 < 0$), $v > 0$ and $v\Delta t < r$ and insert the values afterwards

(c) Calculate the full SFM acceleration of pedestrian 1 for Situation 1 as a function of r and v (as defined above) and insert the numerical values, afterwards. Use following result for the semi-minor axis from Part (b):

$$b\sqrt{r^2 - rv\Delta t}$$

(d) Check if in Situation I the walking pedestrian will “walk through” the standing pedestrian under the simplifying assumption that the contribution of the desired speed can be neglected (by setting $\tau \to \infty$), the target pedestrian does not move away (it may be a static obstacle), and there is no anticipation (which errs on the side of safety)

  (i) Show that, without anticipation and standing Pedestrian j (i.e., for a velocity-independent potential), the sum of the kinetic and potential energies $v^2/2 + \Phi^{\mathrm{int}}$ is conserved

(ii) Calculate the potential once the walking pedestrian 1 has stopped and show that it is smaller than the maximum potential at the pedestrian center;

(iii) Show that, for zero anticipation, the semi-minor axis is equal to the distance $r = -x$ (the elliptic potential degenerates to a circular isotropic potential) and calculate this minimum distance

(e) Now consider Situation II: Pedestrian 1 walks at his/her desired velocity at an angle $\theta = -20\,\text{deg}$ to the x axis (cf the figure) while Pedestrian 2 is still standing at $\vec{x} = (0, 0)$. Would you expect that Pedestrian 1 will swerve to the left or to the right in order to avoid a collision? Give qualitative reasons for your expectation and check your expectation by following lecture formula for the gradient of the interaction potential:

$$-\nabla\Phi^{\text{int}}(\vec{x}) = A \exp\left(\frac{-b}{B}\right) \sqrt{1 + \left(\frac{\Delta\vec{d}}{2b}\right)^2} \left(\frac{\vec{e}_{\vec{x}} + \vec{e}_{\vec{x}+\Delta\vec{d}}}{2}\right).$$

Hint 1: The free terms do not play a role here (why?)

Hint 2: You do not need to evaluate all terms; just evaluating the directionality given by the unit vectors $\vec{e}_{\vec{x}}$ and $\vec{e}_{\vec{x}+\Delta\vec{d}}$ is sufficient

(f) Now consider Situation III which is as Situation II but the target pedestrian 2 walks at $1\,\text{m/s}$ in the negative y-direction. Give your expectation of the direction the subject pedestrian will now swerve to to avoid Pedestrian 2, justify it, and check your expectation by calculating the directionality using above formula

Problem 11.3: Single-file fundamental diagram

see lecture slides