Traffic Flow Dynamics and Simulation

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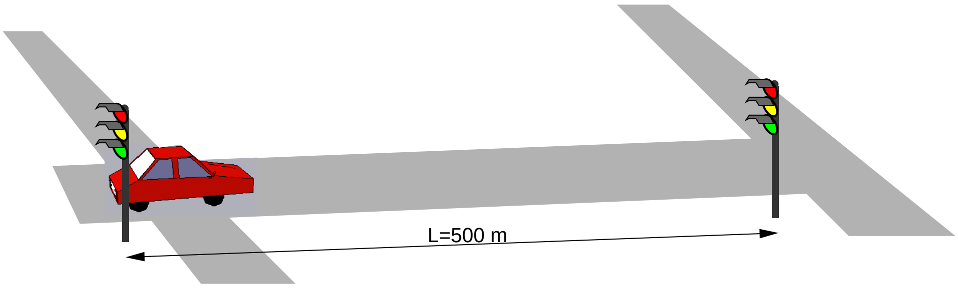
Problem 9.1: Dynamics of a single vehicle between two signalized intersections

A single car in city-traffic conditions (cf. the figure below) can be described by the following time-continuous acceleration model:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \begin{cases} \frac{v_0 - v}{\tau} & \text{if } \Delta v \le \sqrt{2b(s - s_0)}, \\ -b & \text{otherwise.} \end{cases}$$

Here, s denotes the distance to the next car or the next traffic light (whichever is nearer), and Δv is the approaching rate. A red traffic light is modeled by a virtual standing vehicle of zero dimension at the stopping line which is removed when the light turns green.



(a) What is the meaning of the model parameters v_0, τ, s_0, and b? Describe the qualitative acceleration profile after the initially standing car starts moving, and the deceleration profile when approaching a red traffic light. Which essential human property is not taken care of by this model?

(b) The first traffic light turns green at $t = 0\,\mathrm{s}$. Calculate the speed, acceleration, and distance as a function of time for general model parameters assuming that the second traffic light is always green.

(c) Consider now a situation where the subject car is approaching a red traffic light with cruising speed $v_0 = 50\,\mathrm{km/h}$ assuming $s_0 = 2\,\mathrm{m}$ and $b = 2\,\mathrm{m/s^2}$. At which distance to the traffic light does the driver initiate his or her braking maneuver? What is the braking deceleration and the final distance of the standing car to the stopping line?

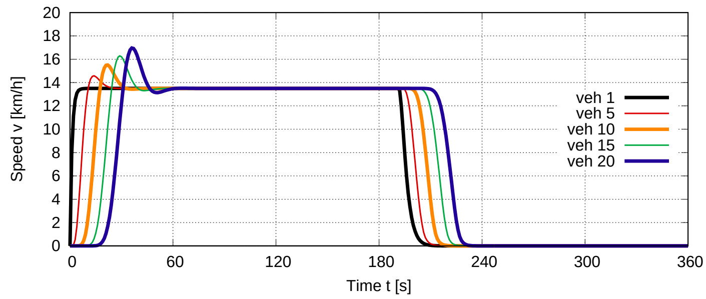
(d) Calculate the trajectory and the speed profile of the car during the complete start-stop cycle for a distance of 500 m between the stopping lines of the two traffic lights assuming $\tau = 5\,\mathrm{s}$ and values for the other model parameters as above.
Hint: There are two phases: The acceleration phase eventually going into cruising mode, and the braking phase. As an essential step, you have to determine the location and the time where the braking maneuver begins. You can assume that the car has already reached its cruising speed at this time.

Problem 9.2: Full Velocity Difference Model

Consider the Full Velocity Difference Model (FVDM) with the triangular fundamental diagram given by

$$\begin{aligned} \frac{\mathrm{d}v}{\mathrm{d}t} &= \frac{v_{\mathrm{opt}}(s)-v}{\tau} - \gamma(v-v_l), \\ v_{\mathrm{opt}}(s) &= \min\left[v_0, \max\left(0, \frac{s-s_0}{T}\right)\right]. \end{aligned}$$

When modeling a platoon of vehicles in an intersection-to-intersection city scenario as in the following figure, neither vehicle reaches, or at least approaches, its cruising speed $v_0 = 54\,\mathrm{km/h}$ although the distance between the traffic light would allow for the cruising speed.



(a) In order to find the underlying mechanism for this, calculate the steady-state final speed if there is a red traffic light (modeled by a standing virtual vehicle) at an arbitrarily large distance

  (i) for general parameters,

  (ii) for the values $v_0 = 54\,\mathrm{km/h}$, $\tau = 5\,\mathrm{s}$, and $\gamma = 0.6\,\mathrm{s}^{-1}$.

  Compare the result with the time series shown in the figure

(b) Identify the model part causing the FVDM to not correctly describe this situation and show that a modification of the above $\gamma(v - v_l)$ term to $\gamma(v - v_l)/\max[1, s/(v_0T)]$ can resolve this inconsistency

Problem 9.3: Reaction to vehicles merging into the lane

A vehicle enters the lane of the considered car causing the gap s to fall short of the equilibrium gap s_e by $50\,\%$. Both vehicles drive at the same speed. Find the resulting (negative) accelerations produced by

(a) the IDM

(b) the simplified Gipps' model

assuming the parameter values $T = \Delta t = 1\,\mathrm{s}$, $a = 1\,\mathrm{m/s^2}$, $b = 2\,\mathrm{m/s^2}$, $\delta = 4$, and $v = v_0/2 = 72\,\mathrm{km/h}$ for all vehicles. (No other parameter values are needed for this problem.)

Problem 9.4: Reactions to a traffic light turning red

A driver approaches a traffic light turning yellow (amber). After his or her reaction and decision time (decision: stop), the driver driving at 54 km/h is 50 m away from the stopping line (modelled as a virtual stopped vehicle of length zero). Calculate the initial braking deceleration for

(a) the IDM

(b) the simplified Gipps' model

with the parameter values $\Delta t = T = 1\,\mathrm{s}$, $s_0 = 2\,\mathrm{m}$, $a = 1\,\mathrm{m/s^2}$, and $b = 2\,\mathrm{m/s^2}$.