Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2021/22, Tutorial No. 8

Problem 8.1: Elasticities

Following table lists the demand for comparable flight connections from three airports for several person groups with different destinations, approach times to the airport, and airfares:

Group /Flight	1: Airport Dresden (DRS)	2: Airport Berlin (BER	3: Airport Frankfurt (FRA)	Choice DRS	Choice BER	Choice FRA
1	30 min, 250 €	120 min, 200 €	300 min, 200 €	35	36	5
2	150 min, 250 €	50 min, 200 €	390 min, 200 €	4	66	2
3	360 min, 250 €	340 min, 200 €	50 min, 200 €	0	2	73
4	120 min, 220 €	120 min, 100 €	120 min, 150 €	2	55	25
5	100 min, 300 €	120 min, 210 €	240 min, 150 €	0	20	24
6	100 min, 200 €	120 min, 250 €	120 min, 300 €	37	12	4

The demand has been modelled with the MNL and following specification for the deterministic utilities:

$$V_{ni}(\vec{\beta}) = \beta_1 \delta_{i2} + \beta_2 \delta_{i3} + \beta_3 T_{ni} + \beta_4 K_{ni}.$$

The model calibration resulted in following estimates:

$$\hat{\beta}_1 = 0.195, \quad \hat{\beta}_2 = 0.581, \quad \hat{\beta}_3 = -0.0130, \quad \hat{\beta}_4 = -0.0244.$$

(a) Discuss the specification: Which kinds of exogeneous variables (characteristics or socioeconomic variables) have been considered ? Are they modelled generically or in an alternative-specific way? Justify the kind of modelling in the context of the problem to be analyzed.

(b) Discuss the parameters and whether their estimated values are plausible. Also discuss why the absolute values of $\hat{\beta}_3$ and $\hat{\beta}_4$ are lower by a factor of ten compared to inner-city modal-split decisions.

(c) Give the global preferences of BER and FRA with respect to DRS in minutes and €. Also calculate the implicit value of time (VoT) in €/h. Finally, give the time equivalent (in minutes) of one utility unit.

(d) Now consider only destination/person group 6. Compare the MNL probabilities of this group for chosing DRS, BER or FRA with the observed percentages.

(e) Calculate for group 6 the microscopic proper elasticities $\epsilon_{6ii}^{(\mathrm{mic,C})}$ and $\epsilon_{6ii}^{(\mathrm{mic,T})}$ of all three airports with respect to the airfares C_{ni} and approach times T_{ni}, respectively.

(f) Now calculate for this group the microscopic airfare cross elasticities $\epsilon_{613}^{(\text{mic,C})}$ und $\epsilon_{623}^{(\text{mic,C})}$. Discuss their values and signs. (i) Prove formally and (ii) discuss intuitively following sum relation for the proper and cross elasticities of one kind (airfares and approach times, in our case):

$$\sum_i P_{ni}\epsilon_{ni3} = 0$$

(g) Give expressions (no calculations required) for the macroscopic airfare proper elasticities $\epsilon_{ii}^{(\text{mac,C})}$ wassuming (i) price changes for all flights by a constant amount, (ii) the same relatve price changes for all flights.

The calculates values are given by

$$\epsilon_{11}^{(\text{mac,C})} = -2.6, \quad \epsilon_{22}^{(\text{mac,C})} = -1.4, \quad \epsilon_{33}^{(\text{mac,C})} = -1.3.$$

Discuss this result.

(h) Give the possible combinations for airfare and approach time at DRS such that this airport can expect the same number of bookings as BER assuming that the airfare at BER is 200 € and the approaching time to BER 2 h. Plot the price *vs.* approach-time relation in a diagram. Why is it irrelevant whether FRA changes its airfares or not?

Problem 8.2: Maximum-Likelihood-Method: the Basics

Consider the simplest possible binary model with just one AC. Then, the choice probability does not depend on the decision maker n and each decision is equivalent to a "Bernoulli experiment"

$$i_{\text{selected}} = \begin{cases} \theta & i = 1 \\ 1-\theta & i = 2 \end{cases}$$

(a) Consider N independent decisions $n = 1, ..., N$.

(i) Verify that the distribution of the sum $N_1 = y$ of decisions for alternative 1 is given by the binomial distribution

$$P(y;\theta) = P_B^{(N,\theta)}(y) = \begin{pmatrix} n \\ y \end{pmatrix} \theta^y (1-\theta)^{N-y}. \tag{1}$$

(ii) Following table shows some evaluated values for $N = 10$:

θ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
$y = 1$	0.39	0.27	0.12	0.040	0.0098	0.0016	0.00014	4.1e-06
$y = 2$	0.19	0.30	0.23	0.12	0.044	0.011	0.0014	7.4e-05
$y = 3$	0.057	0.20	0.27	0.21	0.12	0.042	0.0090	0.00079
$y = 4$	0.011	0.088	0.20	0.25	0.21	0.11	0.037	0.0055
$y = 5$	0.0015	0.026	0.10	0.20	0.25	0.20	0.10	0.026
$y = 6$	0.00014	0.0055	0.037	0.11	0.21	0.25	0.20	0.088
$y = 7$	8.7e-06	0.00079	0.0090	0.042	0.12	0.21	0.27	0.20

Read off the table the ML estimate $\hat{\theta}$ if $y = 2$ and $y = 6$ out of $N = 10$ decisions are in favour of alternative 1

(iii) Give the analytical likelihood function $L(\theta)$ for a given number y of decisions for $i = 1$ and perform the ML estimation using either the likelihood or the log-likelihood function

(b) The number of incoming telephone calls during a certain hour is modelled by the Poisson distribution (what are the assumptions for this to be valid?)

$$P(y, \beta_1) = P(y, \mu) = \frac{\mu^y e^{-\mu}}{y!}$$

Some values are given in following table:

μ	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
$y = 1$	0.37	0.33	0.27	0.21	0.15	0.11	0.073	0.050
$y = 2$	0.18	0.25	0.27	0.26	0.22	0.18	0.15	0.11
$y = 3$	0.061	0.13	0.18	0.21	0.22	0.22	0.20	0.17
$y = 4$	0.015	0.047	0.090	0.13	0.17	0.19	0.20	0.19
$y = 5$	0.0031	0.014	0.036	0.067	0.10	0.13	0.16	0.17

Employee A received two calls, and B, answering a different sort of calls, received four. Give the analytical log-likelihood parameterized for the numbe rof incoming calls and perform the ML estimation (i) using the table, (ii) using the aalytic likelihhod function, and (iii) using the log-likelihood.

(c) The weekly sales Y of a public transport company (quasi-continuous) can be assumed to be Gaussian (what are the conditions for this to be a good approximation?). For the actual sales Y_i in a given week i, we assume $Y_i \sim i.i.d.N(\mu\sigma^2)$ (discuss when this is a good approach!), i.e., the density is given by

$$f(y|\vec{\beta}) = f(y|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}.$$

(i) Give the log-likelihood $\tilde{L}(\mu, \sigma^2)$ for observations $\vec{y} = (y_1, ..., y_n)'$ in week 1 till week n, respectively. Assume as usual, that for independent events, the combined multivariate probability density is the product of the single densities.

(ii) Perform the ML estimation and compare the result with the OLS estimate for the trivial regression model $y = \mu + \epsilon,\ \epsilon \sim N(0, \sigma^2)$.