Traffic Flow Dynamics and Simulation

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Problem 7.1: Dissolving queues at a traffic light

In the triangular fundamental diagram, the velocity of moving downstream fronts of congested traffic is given by the wave velocity $c = -l_{\text{eff}}/T$. Try to intuitively understand this relation for a queue of standing vehicles behind a traffic light after it turns green. Total waiting time during one red phase of a traffic light

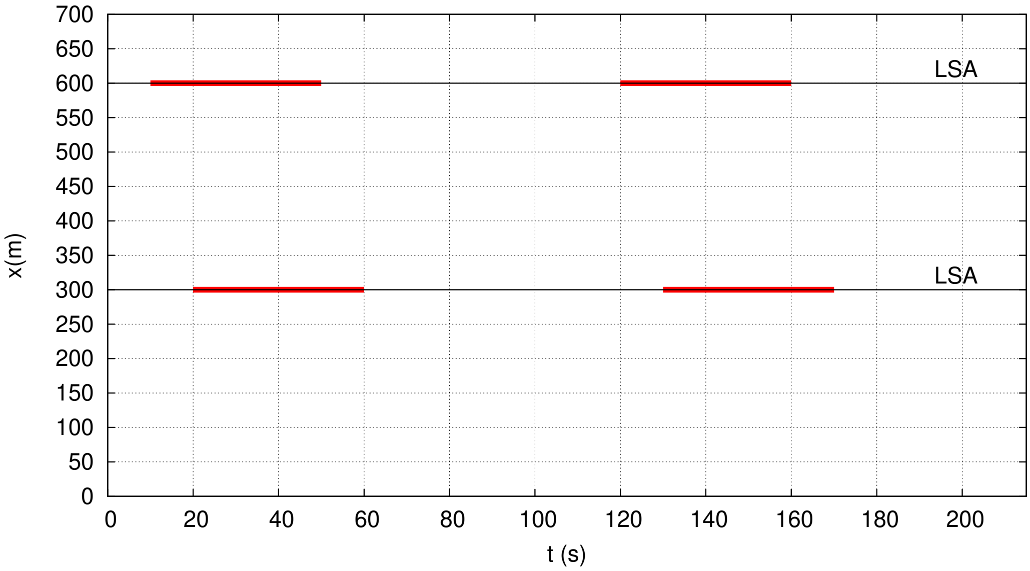
Problem 7.2: Total waiting time during one red phase of a traffic light

Calculate the total waiting time of all vehicles caused by one red phase of duration τ_r for the LWR model with a triangular fudamental diagram. Assume the conditions of the example in Section **??**, i.e., a constant inflow Q_{in}, and sufficient capacity to avoid over-saturation, i.e., the queue completely dissolves before the next red phase. Express the solution as a function of τ_r, the maximum density, the propagation velocities c^{up} of the upstream boundary, and the universal propagation velocity c^{cong} of congested traffic.

Problem 7.3: Two consecutive signalized intersections: green waves

The traffic flow on one direction of a mainroad with two signalized intersections shell be modelled with a LWR model with tridiagonal fundamental diagram $Q(\rho) = \min\left(V_0\rho, 1/T(1 - l_{\text{eff}}\rho)\right)$ parameterized to $v_0 = 54\,\text{km/h}$, $T = 1.5\,\text{s}$ and $l_{\text{eff}} = 7.5\,\text{m}$.

The following figure shows the locations of the traffic lights and the durations and relative offset of their red phases:



(a) Determine the free-road capacity (without traffic lights) and the critical density ρ_c “at capacity”.

(b) Calculate the traffic density in the waiting queues and the propagation velocity of perturbations inside the queue. Also argue that the same propagation velocity also describes the downstream front of the queu once the light turned greem.

(c) Assume now a constant demand of 1029 veh/h. Under which conditions this is a plausible assumption? Furthermore, assume a free-flow capacity of $Q_{\max} = 1\,800\,\text{veh/h}$ and determine the cycle-averaged capacity. Is it enough for the given demand?

(d) Calculate the spatiotemporal dynamics of the queue caused by the first red phase of the upstream traffic light at 300 m assuming a constant inflow of unit[1029]Fz/h. When and where the queue will dissolve?

(e) Assume now following propagation velocities:
   - Queue → outflow and outflow → queue: $-5\,\text{m/s}$,
   - inflow-queue: $-2.5\,\text{m/s}$, and
   - outflow-inflow and inflow-outflow: $15\,\text{m/s}$. Draw into the diagram the spatiotemporal dynamics of all queues, outflow regions, and empty regions assuming initially free traffic with a flow equal to the demand everywhere.

Problem 7.4: Flow instability in Payne’s model and in the Kerner-Konhäuser Model

For the following models, the following simple condition for flow stability is valid:

$$\left(\rho V_{\mathrm{e}}'(\rho)\right)^2 \leq P'(\rho).$$

This applies if the density gradient terms of the macroscopic model can be written as a “pressure gradient” $-\frac{1}{\rho}P'(\rho)\frac{\partial\rho}{\partial x}$ at the right-hand side.

Consider Payne’s model and the Kerner-Konhäuser Model with a triangular fundamental diagram

$$Q_e(\rho) = \begin{cases} V_0\rho & \text{if } \rho \le \rho_{\text{crit}} \\ \frac{1}{T}\left[1-\rho(s_0+l)\right] & \text{falls } \rho_{\text{crit}} < \rho \le \rho_{\text{max}}, \end{cases}$$

and the parameters $l_{\text{eff}} = 6\,\text{m}$, $V_0 = 144\,\text{km/h}$ and $T = 1.1\,\text{s}$.

(i) Show that Payne’s model with the speed equation

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e(\rho)-V}{\tau} + \frac{V_e'(\rho)}{2\rho\tau}\frac{\partial \rho}{\partial x}$$

is unconditionally linearly stable if $\tau < T/2$, and flow unstable in the congested regions, otherwise.

(ii) Now consider the Kerner-Konhäuser Model with the speed equation

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e(\rho)-V}{\tau} - \frac{\theta_0}{\rho}\frac{\partial \rho}{\partial x} + \frac{\eta}{\rho}\frac{\partial^2 V}{\partial x^2}.$$

Determine the parameter θ_0 such that this model is string unstable in the density range $\rho \in [20\,\text{vehicles/km}, 50\,\text{vehicles/km}]$.