Traffic Flow Dynamics and Simulation

SS 2024, Tutorial 5, page 1

Problem 5.1: Fundamental diagram as reconstructed from stationary detector data and traffic waves

As already pointed at in Tutorial 4.2 and Lecture 3, one can use stationary detectors at several locations (cf. the time series of the figure on the next page) to directly estimate the wave speed and thus avoid systematic errors in estimating the triangular fundamental diagram parameters T and ρ_{max},

$$Q(\rho) = \begin{cases} V_0\rho & \rho \le \rho_c \\ \frac{1}{T}\left[1 - \frac{\rho}{\rho_{\text{max}}}\right] & \rho > \rho_c \end{cases} \tag{1}$$

As we will derive in the lectures ahead, the wave velocity of congested traffic waves is given in terms of ρ_{max} and T by

$$w = Q'(\rho) = -\frac{1}{T\rho_{\text{max}}}. \tag{2}$$

(a) Estimate w from the spatiotemporal traffic state reconstruction depicted on the next page (the reconstruction itself has been treated in Lecture 04).

(b) Estimate w from the speed or flow time series on p.2. Argue that the estimation of w from the speed time series is unbiased even if the speed estimation itself is biased.

(c) Estimate the capacity $\hat{Q}_{\text{max}}$ by approximating Q_{max} to the outflow of the congested traffic as measured in the non-congested regions.

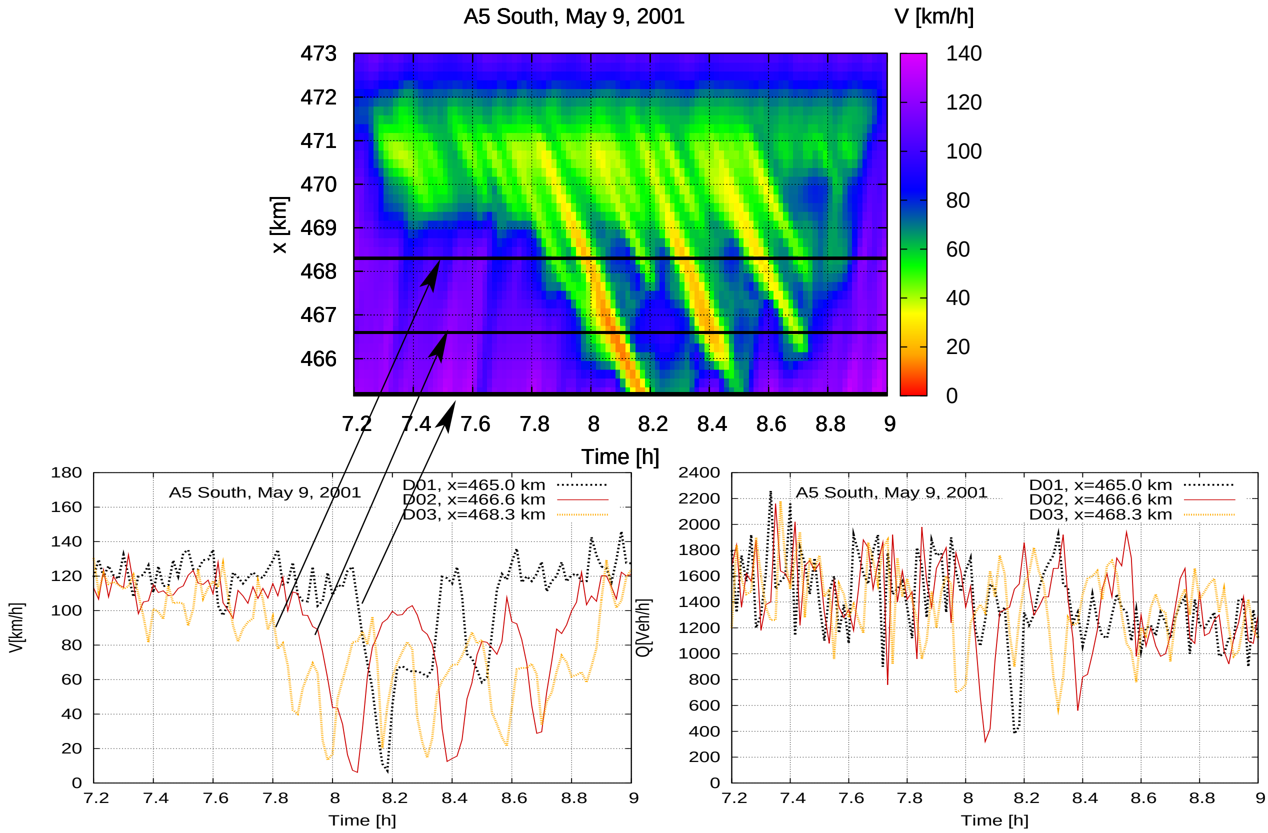
(d) Give the theoretical capacity Q_{max} of the triangular fundamental diagram

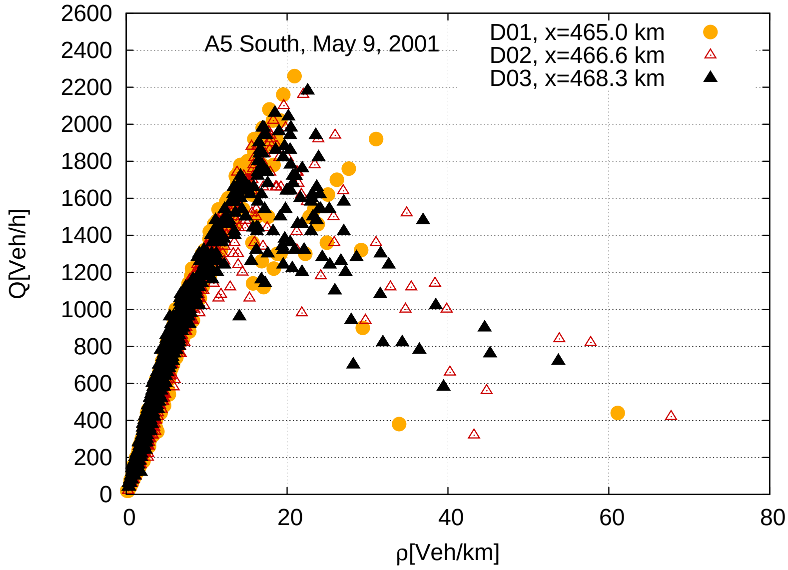
$$Q(\rho) = \max[V_0\rho, 1/T(1 - \rho/rho_{\text{max}})] \tag{3}$$

in terms of v_0, T und ρ_{max}.

(e) Estimate v_0 from the time series and use Eq. (3) and (2) to obtain an unbiased estimate for ρ_{max} and T.

(f) For comparison, estimate v_0, T, and ρ_{max} from the flow-density scatter plot of the relevant detectors: Discuss and explain the quantitative difference.


A5 South, May 9, 2001
V [km/h]
x [km]
Time [h]
A5 South, May 9, 2001
D01, x=465.0 km
D02, x=466.6 km
D03, x=468.3 km
V[km/h]
Time [h]
A5 South, May 9, 2001
D01, x=465.0 km
D02, x=466.6 km
D03, x=468.3 km
Q[Veh/h]
Time [h]


A5 South, May 9, 2001
D01, x=465.0 km
D02, x=466.6 km
D03, x=468.3 km
Q[Veh/h]
ρ[Veh/km]

Problem 5.2: Marathon

In order to avoid congestions among the athletes with ensuing critical situations, the managers of a city Marathon event assess a new course. The starting field and the first 8 km consist of roads and places that are wide enough for the risk of jams to be marginal. After kiloemeter 8, however, the course includes a 5 m wide underpass which has been identified as a bottleneck. In order to assess the risk and, if applicable, counter it by a limitation of the number of athletes and/or a wave start, the two-dimensional directed flow of athletes is modelled macroscopically by a 2d-LWR model with following fundamental diagram relating the flow density J with the 2d density ρ:

$$J(\rho) = \begin{cases} V_0 \rho & \rho \le \rho_c \\ J_0 \left[1 - \frac{\rho}{\rho_{\text{max}}}\right] & \rho > \rho_c \end{cases}$$

The parameters of the congested branch, assumed to be the same for all local populations of athletes, have been estimated by

$$J_0 = 2.5\,(\text{ms})^{-1}, \quad \rho_{\text{max}} = 3\,\text{runners/m}^2$$

(a) Determine the specific capacity (maximum flow density) J_{max} as a function of V_0, J, and ρ_{max}. Calculate the numerical value for slow (8 km/h), average (11 km/h) or very fast (15 km/h) athletes. Do the values differ by much?

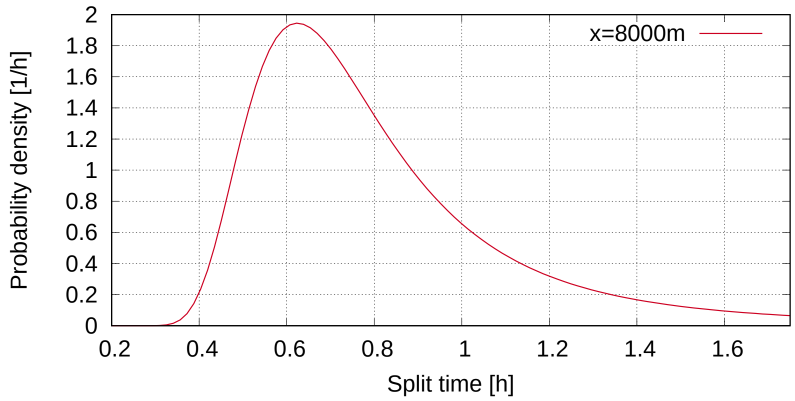
(b) Determine the capacity (maximum throughflow) of the underpass for a population of average (11 km/h) athletes

(c) Based on past events, the event managers calculate with a Gaussian speed distribution of the athletes according to

$$V \sim N(\mu, \sigma^2), \quad \mu = 3\,\text{m/s}, \ \sigma^2 = 1\,(\text{m/s})^2.$$

Additionally, the fast athletes remain fast and the slow ones slow such that the individual speed (provided there are no congestions) is constant throughout their respective race. Calculate the density function $f(\tau)$ of the split time τ at the bottleneck (the split time is the time interval after the start at which an athlete passes a certain location). To do so, neglect the initial length of the starting field (fastest runners first) compared to the length this field has extended to at the bottleneck location

(d) The distribution function of the split times for a mass start is now given by following figure:



Determine the maximum number N of participants for which there will be no congestion at the bottleneck using the figure below.

(e) Can this number be increased if there is a wave start consisting of a fast and a slow wave of $N/2$ athletes, each ($\sigma^2_{\text{fast}} = \sigma^2_{\text{slow}} = 0.5\,(\text{m/s})^2$, 10 minutes delay between the waves)? Use the figure below where all densities are already plotted.