Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2023/24, Solutions to Tutorial No. 11

Solution to Problem 11.1: Combined destination and mode choice: Nested-Logit Model

(a) The last person took 5 recorded shopping trips. In total, there are 10 persons taking between 3 and 5 errands resulting in a total number of

$$N = \sum_{n=1}^{10} (y_{n11} + y_{n12} + y_{n21} + y_{n22}) = 44$$

errands, or equivalently, combined decisions.

(b)
- *Top-level destination choice:* The destination is mainly determined by the fridge fill level (rather low, prefarably go to the discount store; rather high: "Aunt Emma" is enough).
- *Lower-level mode choice:*. The conditional mode choice for a chosen destination depends on the travel times.
- *Coupling between the levels:* This is mediated by the inclusion variables essentially indicating the overall attractivity of given nest which, of course, depends mainly on the travel times. More specifically, $\lambda_l I_l$ gives the attractivity of nest l as seen from the top-level. If $\lambda \to 0$, this is identical to the lower-level utility of the "best" mode, otherwise, it is somewhat higher than the "best" mode reflecting the fact that the very presence of a choice generates some additional utility.

(c)
- Parameter β_1, β_3: Zeitsensitivitäten beider Modi bei der Fahrt zur Tante Emma bzw zum Discounter. Beide sollten negativ sein. Innerhalb jedes Nests werden die Fahrtzeiten generisch modelliert, die Zeitsensitivitäten von ÖV und MIV sind also gleich (aber die gemeinsame Sensitivität ist i.A. unterschiedlich für Tante Emma und den Discounter).
- Parameter β_2: AC, Attraktivitätsvorteil für den ÖV gegenüber dem MIV als Referenzalternative beim Einkauf bei Tante Emma. Da dort nicht so viel eingekauft wird, muss man nicht viel schleppen und der ÖV ist bei gleichen Zeiten meist attraktiver, also wird $\beta_2 > 0$ erwartet.
- Parameter β_4: AC, Attraktivitätsvorteil für den ÖV gegenüber dem MIV als Referenzalternative beim Einkauf beim Discounter. Da man beim Discounter eher viel einkauft, ist ein Auto praktischer und β_4 wird deshalb negativ erwartet.

(d) For Person $n = 10$, we have within the nests

$$\frac{\tilde{V}_{10,1,1}}{\lambda_1} = 25\hat{\beta}_1 + \hat{\beta}_2 = -3.73,$$
$$\frac{\tilde{V}_{10,1,2}}{\lambda_1} = 10\hat{\beta}_1 = -1.84,$$
$$\frac{\tilde{V}_{10,2,1}}{\lambda_2} = 25\hat{\beta}_3 + \hat{\beta}_4 = -7.66,$$
$$\frac{\tilde{V}_{10,2,2}}{\lambda_2} = 20\hat{\beta}_3 = -5.79.$$

The conditional choice probabilities inside the nests are

$$P_{nm}|l = \frac{e^{\tilde{V}_{nlm}/\lambda_l}}{N_{nl}}, \quad N_{nl} = e^{\tilde{V}_{nl1}/\lambda_l} + e^{\tilde{V}_{nl2}/\lambda_l},$$

i.e.,

$$P_{10,1}|1 = \frac{e^{-3.73}}{e^{-3.73} + e^{-1.84}} = 0.132,$$
$$P_{10,2}|1 = \frac{e^{-1.84}}{e^{-3.73} + e^{-1.84}} = 0.868,$$
$$P_{10,1}|2 = \frac{e^{-7.66}}{e^{-7.66} + e^{-5.79}} = 0.134,$$
$$P_{10,2}|2 = \frac{e^{-5.79}}{e^{-7.66} + e^{-5.79}} = 0.866$$

The realized relative frequencies are

$$f_{10,1}|1 = 0, \quad f_{10,2}|1 = 1, \quad f_{10,1}|2 = 1/4, \quad f_{10,2}|2 = 3/4.$$

indicating a decent degree of fitting (see the plots in the lecture set of slides)

(e) Generally, the inclusion values are given by

$$I_{nl} = \ln\left(e^{\tilde{V}_{nl1}/\lambda_l} + e^{\tilde{V}_{nl2}/\lambda_l}\right) = \ln N_{nl}.$$

For Person 10, we have the two inclusion values

$$I_{10,1} = -1.70, \quad I_{10,2} = -5.65,$$

Meaning:

The inclusion values I_l denote the scaled utility of nest l *as seen from the top-level point of view*. In the units used at top-level, the nest utilities are given as $\lambda_l I_l$. They can be interpreted as *the expectation value of the largest total utility of an alternative inside the nest*

- Since the random utilities sometimes make an alternative with a lower deterministic utility the favourite and the *Homo Oeconomicus* always choses the alternative with the highest total utility, the nest utility is generally greater than the highest deterministic utility: If all alternatives inside a nontrivial nest have the same determinsitic utility, the nest utility exceeds this deterministic utility by the expecation value of the maximum of the random utilities (minus the expectation value of a single random utility) which is always > 0
- If the random utilities are highly correlated inside the nest, λ is small and $\lambda_l I_l$ is essentially equal to the maximum in-nest determinsitic utility.
- Independently from λ, this is also true if one in-nest alternative dominates all others
- Of course, for as trivial nest, the inclusion value is equal to the determinsitic utility of the single nest alternative,

(f)
- Parameter β_5: shift of preference towards "Aunt Emma" with increasingg fridge fill level. $\beta_5 > 0$ expected since, with a full fridge, a bigger store becomes increasingly unattractive (and one *must* go shopping in this specification)
- Parameter β_6: Preference "Tante Emma" *vs.* discount store for an empty fridge and the same in-nest utilities $\lambda_l I_l$ (essentially the travel times). $\beta_6 < 0$ expected unless the surveyed persons have a strong bias towards small stores
- λ_1, λ_2: The correlation parameters are inside their valid ranges [0,1] (deviations within the estimation error are, of course, possible and would be harmless). Since the correlation parameters are nearer to zero than to one, the random utilities (RUs) inside the nest are small compared to that outside (ratio $\lambda/(1-\lambda)$) and the total RUs are highly correlated for the same nest (correlation $r = 1 - \lambda_l^2 > 0.9$).

It is also possible to describe the correlation structure inside the nest with a single correlation parameter λ. Then, the top-level deterministic utility would be specified as

$$W_l = \beta_5 F \delta_{l1} + \beta_6 \delta_{l1} + \lambda I_l$$

(g) As is the case for the conditional probabilities inside the nests, the top-level choice of the nest is modelled by a normal Logit model:

$$P_{nl} = \frac{e^{W_{nl} + \lambda_l I_{nl}}}{e^{W_{n1} + \lambda_1 I_{n1}} + e^{W_{n2} + \lambda_2 I_{n2}}}$$

For Person $n = 10$, we have

$$W_{10,1} = \hat{\beta}_5 F_{10} + \hat{\beta}_6 + \hat{\lambda}_1 I_1 = \hat{\beta}_6 + \hat{\lambda}_1 I_1 = -2.31, \quad W_{10,2} = \hat{\lambda}_2 I_2 = -1.20$$

leading to the top-level probabilities for the two nests

$$P_{10,1} = \frac{e^{W_{10,1}}}{e^{W_{10,1}} + e^{W_{10,2}}} = 0.248, \quad P_{10,2} = \frac{e^{W_{10,2}}}{e^{W_{10,1}} + e^{W_{10,2}}} = 0.752$$

The corresponding observed percentages are $f_1 = 1/5$ und $f_2 = 4/5$.

Finally, the combined probabilities for the combined decision "go to shop type l using driving mode m is given by

(see also the plots in lecture slides)

$$P_{10,1}P_{10,1|1} = 0.033, \quad P_{10,1}P_{10,2|1} = 0.216, \quad P_{10,2}P_{10,1|2} = 0.101, \quad P_{10,2}P_{10,2|2} = 0.651,$$

while the corresponding observed percentages are 0/5, 1/5, 1/5 und 3/5, respectively.

(h) For both $\lambda_l = 1$, the nested logit model reverts to a normal MNL with the deterministic utilities

$$V_i = W_l + \tilde{V}_{lm}$$

With the mapping $i = 1, ..., 4$ to the combinations $(l, m) =$ (1,1), (1,2), (2,1) und (2,2), respectively, the factor with β_1 must be restricted to nest 1 (selector $\delta_{i1} + \delta_{i2}$) and that of β_2 to the alternative $i = 1$ (selector δ_{i1} similar selectors apply for the other nest and for the top-level factors proportional to β_5 and β_6 leading to the given MNL utility function.

Weakness of the MNL: It does not reflect unobserved preferences for a shop type which, in our example, nearly make up the whole random utility, i.e., the unobserved preferences for a driving mode are lower than that for the shop type.

Test if the NL model is needed or if a MNL is enough: Likelihood-ratio test with the MNL being the NL restricted to $\lambda_1 = \lambda_2 = 1$. If H_0: "The MNL is equally suited" applies, the LR test statistic is $\sim \chi^2(2)$ distributed.

In the same way, one could also test a model variant with common in-nest correlations (restriction $\lambda_1 = \lambda_2$, test statistic $\sim \chi^2(1)$ if H_0)

(i) Add a third trivial nest $l = 3$ "stay at home" to the top-level choice. The top-level utility component is augmented by an AC and a fridge-level sensitivity:

$$W_l = \beta_5 F \delta_{l1} + \beta_6 \delta_{l1} + \lambda_1 I_l \delta_{l1} + \lambda_2 I_l \delta_{l2} + \beta_7 F \delta_{l3} + \beta_8 \delta_{l3}$$

- β_7: preference shift in favour of "stay at home" with rising fridge fill level. Should be even more positive than β_5
- β_8: A general "laziness factor": The lazier a person is, the lower the number of shopping trips (regardless of the fridge level), and the higher the preference β_8 for staying at home.