Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2023/24, Tutorial No. 10

Problem 10.1: Likelihood-ratio test

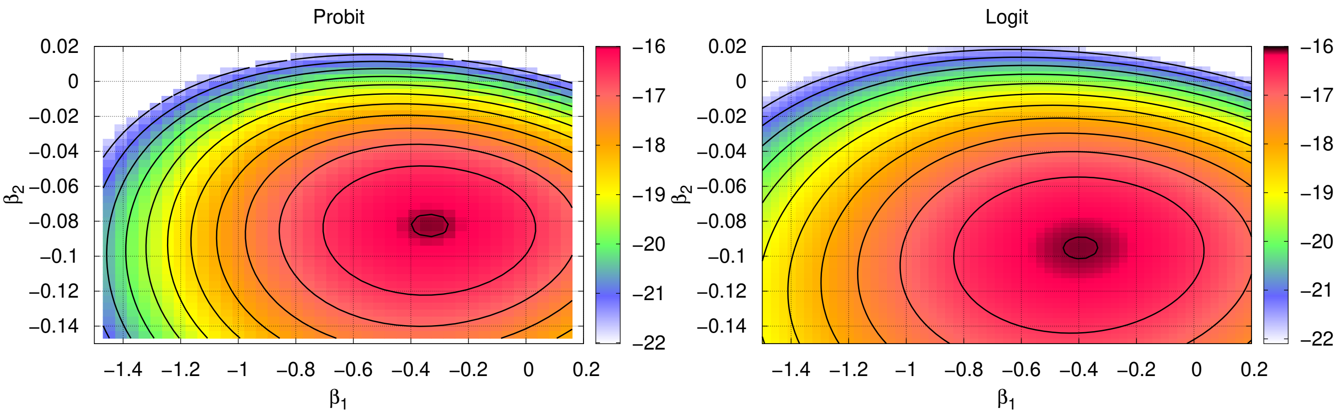
Given are following *Revealed-Choice* data:

Person	Time Alternative 1 ped/bike [min]	Time Alternative 2 motorized [min]	Choice Alt. 1	Choice Alt. 2
1	15	30	4	1
2	10	15	1	4
3	20	20	3	2
4	30	25	2	3
5	30	20	1	4
6	60	30	0	5

They should be analyzed by binary Logit and Probit models with following specification of the deterministic utilities:

$$V_{ni}(\beta_1, \beta_2) = \beta_1 \delta_{i1} + \beta_2 T_{ni}$$

Following contour plots give the loglikelihood $\tilde{L}(\beta_1, \beta_2)$ for both models (distance of 0.5 between the contour lines):

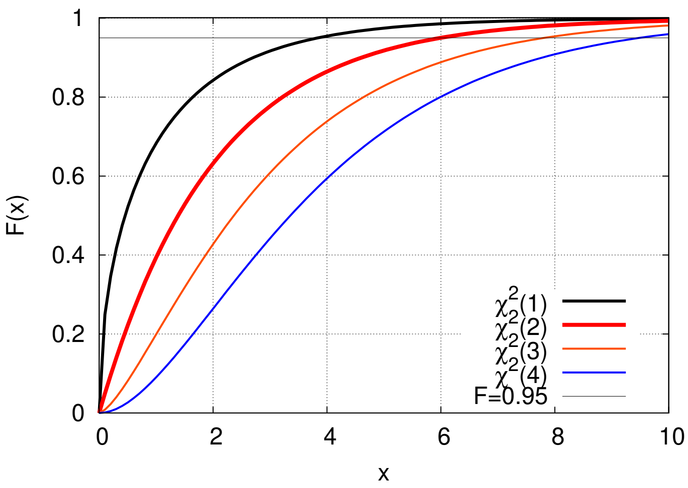


(a) Test, at an error probability of 5 %, whether there is a significant ad-hoc preference of one transportation mode over the other by performing LR-tests of the "time-only" specification $V_{ni} = \beta_2 T_{ni}$, and the full specification both for the Logit and Probit models.

Hint: Use the contour plots and the graphs of some χ^2 distributions below.

(b) Test whether the time sensitivity is a relevant factor by comparing the AC-only with the full models

(c) Compare the full, "AC-only", and "time-only" models with the trivial model $V_{ni} = 0$ with appropriate LR tests.



Problem 10.2: Likelihood-ratio test for regression models: $\lambda = T^2$

The LR model can also be applied to regression models provided that the distribution of the random term ϵ is known

(a) Why this is only possible if the distribution is known?

(b) Give the ML estimation of the one-parameter "constant-only regression model" M:

$$y = \beta_0 + \epsilon, \quad \epsilon \sim i.i.d.N(0, \sigma_\epsilon^2)$$

given data $\{(x_i, y_i)\}$, $i = 1, ..., n$.

(c) Now consider the LR-test between model M and the parameterless fixed model M_0: $y = \mu_0 + \epsilon$ with a fixed constant μ_0. Furthermore, assume that there is a sufficient amount of data such that the estimation error of $V(\epsilon) = \sigma^2$ can be neglected, so σ^2 is known. Show that, in this case, the test variable $T = 2[\ln L(\hat{\beta}_0) - \ln L_0]$ (with L_0 the log-likelihood of M_0) exactly obeys $T \sim \chi^2(1)$.

Hint: a sum of m i.i.d squares of standardnormal distributed random variables is $\chi^2(m)$ distributed.

(d) Show that this LR test is exactly equivalent to a t-test for known variance of the null hypothesis H_0: $\beta_0 = \mu_0$.