Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2023/24, Tutorial No. 9

Problem 9.1: Estimation of trivial and AC-only models

(a) Show that, for the trivial utility function $V_{ni} = 0$, the binomial Logit and Probit trivial models have a choice probability of $P_1 = P_2 = 1/2$. Generalize this result to the MNL.

(b) Show that, for general discrete-choice models with the likelihood function

$$\ln L(\vec{\beta}) = \sum_{n=1}^{N} \sum_{i=1}^{I} y_{ni} \ln P_{ni}(\vec{\beta}), \tag{1}$$

i.e., independent decisions[1], the ML estimation of the AC0-only model $V_{ni} = \sum_{m=1}^{I-1} \beta_m \delta_{mi}$ leads to estimated probabilities

$$\hat{P}_i = \frac{N_i}{N}, \quad N_i = \sum_n y_{ni},$$

i.e., the probabilities are equal to the percentaged total choice number for a certain alternative i. *Hint:* Lagrange multipliers. Also use Lagrange mutlipliers to show that, for general trivial models with Log-likelihoods according to (1) (i.e., uncorrelated random utilities), the calibrated choice probabilities are given by $P_i = 1/I$. Finally show that, for the binary logit model with $V_{ni} = \beta_1 \delta_{i1}$, the ML estimation of the parameter itself is given by

$$\hat{\beta}_1 = \ln \left(\frac{N}{N_i} - 1 \right).$$

Problem 9.2: Considerations of a car salesman

A car salesman takes much effort in customer service and free test drives with new cars although most customers will not eventually buy a car. Sometimes, he has the impression that people just want to test the newest models for free. In order to concentrate on people seriously interested in buying a new car, he shrewdly obtained, during the negotiations, the age of the customer's present car (if any) and whether it has been bought as a new or used car. Furthermore, he records the discounts offered to each customer and whether the negotiations were successful.

[1] Within the alternatives, the random utilities (RUs) need *not* to be i.i.d. or even uncorrelated . However, for highly heteroskedastic and/or corrrelated RUs, the calibrated choice probabilities may not be accessible in some cases, i.e., there are no parameter values to reach them.

Age of present car (years)	1	3	5	7	10	5	8	10	12	14
Discount (1 000 €)	1	2	3	1	0	3	2	2	1	3
Present car bought new (0=N, 1=Y)	0	0	1	0	1	1	1	1	0	0
Successful deal (0=N, 1=Y)	0	0	0	0	0	1	1	0	0	1

The data are analyzed with a binomial Logit model specified as

$$V_{ni} = \beta_1 \delta_{i1} + \beta_2 T_n \delta_{i1} + \beta_3 R_n \delta_{i1} + \beta_4 \delta_{i1} \begin{cases} 1 & \text{present car bought new} \\ 0 & \text{others} \end{cases}$$

where $i = 1(2)$ denotes a successful (unsuccessful) deal, respectively, T_n is the age of the present car (assuming there is one), and R_n is the discount offered (in 1 000 €).

(a) Is this a stated or revealed-choice query?

(b) Enumerate all characteristica, socio-economic variables, and ACs.

(c) Discuss if the model is well specified for a generic instead of an alternativ-specific ansatz for the present car age, i.e., the relevant factor is given by $\Delta V_{ni} = \beta_2 T_n$ instead of $\beta_2 T_n \delta_{i1}$.

(d) (i) Give general expressions for the four realized and modelled property sums which must be equal after a ML estimation

(ii) Give the numerical values for the realized property sums X_m^{data} and the property sums X_m^{mod} estimated from the Logit model with $\vec{\beta} = \vec{0}$.

(e) The ML estimation resulted in

$$\hat{\vec{\beta}} = (-9.2, 0.35, 2.2, 1.3)'$$

Explain/justify why $\hat{\beta}_1$ takes on such a negative value.

(f) A new customer already has a five-year old car then bought as a new car. The salesman offers a final discount of 2 000 €. Give the probability that the customer will make the deal (buy a new car).

(g) In the present specification, customers having no present car cannot be included (why?). Generalize the model specification to include customers potentially buying his/her first car.