Variance-driven traffic dynamics

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Abstract. We investigate the adaptation of the time headways in car-following models as a function of the local velocity variance, which is a measure of the inhomogeneity of traffic flows. We apply our *meta-model* to several car-following models and simulate traffic breakdowns in open systems with an on-ramp bottleneck. Single-vehicle data generated by 'virtual detectors' show a semi-quantitative agreement with microscopic data from the Dutch freeway A9. This includes the observed distributions of the net time headways and times-to-collisions for free and congested traffic, and the velocity variance as a function of traffic density. Macroscopic properties such as the observed wide scattering of flow-density data are reproduced as well, even for deterministic simulations. We explain these phenomena by a self-organized variance-driven process that leads to the spontaneous formation and decay of long-lived platoons.

1 Introduction

One of the open questions of traffic dynamics is a microscopic understanding of the observed wide variation of the time headways which is closely related to the wide scattering of flow-density data in the congested traffic regime (see, e.g., Refs. [1–3] for an overview). Moreover, the most probable value of the time headway in congested traffic is larger by a factor of about 2 compared to free traffic, see Fig. 1(a).

With the increasing availability of single-vehicle data, further statistical properties of traffic became the subject of investigation such as the velocity variance as a function of the traffic density [4], or the distribution of the times-to-collision (TTC), which is surprisingly invariant with respect to density changes (compared to distance, time gap, or velocity distributions), see Fig. 1(b).

In this contribution, we propose a variance-driven adaptation mechanism (VDT mechanism), according to which drivers increase their time gaps T when the local traffic dynamics is unstable or largely varying.

2 Variance-driven time headways (VDT)

We will formulate the VDT mechanism in terms of a *meta-model* applicable to any car-following model containing the 'safe' time headway or a related parameter such as the desired (equilibrium) distance. Some examples are the optimal-velocity model (OVM) [5], the velocity-difference model (VDIFF) [6], the intelligent-driver model (IDM) [7], or the Gipps model [8].

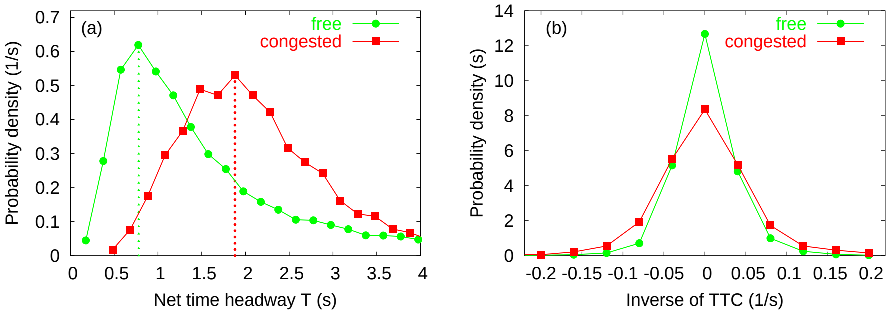

Fig. 1. Empirical statistical properties of cars following any kind of vehicle obtained from single-vehicle data from the left lane of the Dutch freeway A9 from Haarlem to Amsterdam. (a) Net time headway; (b) Inverse times-to-collision, for two traffic situations: The data set for 'free traffic' includes all single-vehicle data where the one-minute average of velocities was above 20 m/s, and the traffic flow above 1000 vehicles/h. 'Congested traffic' includes all data where the one-minute average of the velocities was below 15 m/s.

We assume that smooth traffic flow allows for lower values of the time headway than disturbed traffic flow, i.e., the actual time headway

$$T = \alpha_T T_0 = \min\left(\alpha_T^{\max}, 1 + \gamma V_n\right). \tag{1}$$

is increased in nonperturbed traffic with respect to the minimum time headway T_0 by a factor $\alpha_T \geq 1$. Furthermore, we characterize disturbed traffic flow (such as stop-and-go traffic) by the *local variation coefficient*

$$V_n = \frac{\sqrt{\theta_n}}{\bar{v}_n}, \tag{2}$$

where the local velocity average $\bar{v}_n$ and velocity variance θ_n, are calculated from the own velocity v_i and the velocities of the $(n-1)$ predecessors $(j-i)$:

$$\bar{v}_n = \frac{1}{n}\sum_{j=0}^{n-1} v_{j-i}, \qquad \theta_n = \frac{1}{n-1}\sum_{j=0}^{n-1}(v_{j-i} - \bar{v}_n)^2. \tag{3}$$

The VDT has three parameters, namely the number n of vehicles to determine the local velocity variance, the maximum multiplication factor $\alpha_T^{\max}$ by which the time headway is increased compared to perfectly smooth traffic, and the sensitivity γ. The factor $\alpha_T^{\max}$ can be estimated from empirical time-headway distributions as the ratio of the most probable time headways for congested and free traffic, respectively, while the sensitivity γ is calibrated to the empirical variation coefficient $V_n = V_n(\rho)$ as a function of density (Fig. 2). Throughout this contribution, we will assume the values $n = 5$, $\alpha_T^{\max} = 2.2$, and $\gamma = 4$.

Notice that, in the special case $n = 2$, the local variation coefficient for vehicle i is given by $V_2^{(i)} = \sqrt{2}|v_i - v_{i-1}|/(v_i + v_{i-1})$, i.e., the VDT mechanism adds a contribution to the underlying car-following model which is proportional to the velocity difference to the immediate predecessor. For $n > 2$, the VDT includes some anticipation beyond this vehicle which is expected to be an essential ingredient for human driving [9].

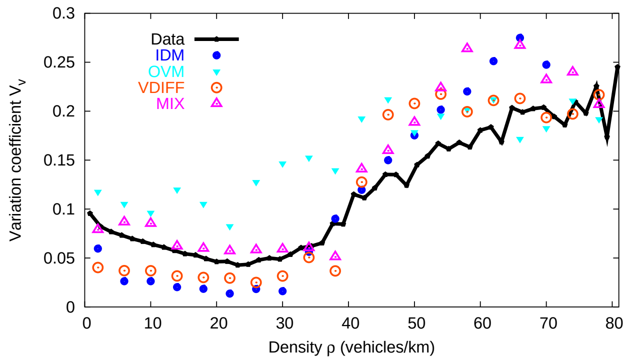

Fig. 2. Velocity variation coefficient $\sqrt{\theta}/\langle v_i \rangle$ as a function of overall traffic density from single-vehicle data of the Dutch freeway ("Data"), and from a virtual detector 4 km upstream of the on-ramp, when the VDT is simulated with various underlying models.

2.1 Acceleration noise

Since the VDT is essentially based on fluctuations of the velocity, it is to be expected that purely deterministic underlying models yield unrealistic results due to the lack of an initial source triggering fluctuations. For simplicity, we will just add a white (independent and δ-correlated) noise term [10] to the deterministic car-following acceleration $a_i^{(\mathrm{det})}$ according to

$$\dot{v}_i = a_i^{(\mathrm{det})}(t) + \sqrt{Q}\,\xi_i(t). \qquad (4)$$

Here, Q denotes the fluctuation strength (we will assume $Q = 0.1\mathrm{m}^2/\mathrm{s}^3$ for all simulations unless stated otherwise), and the white noise $\xi_i(t)$ is assumed to be unbiased and δ-correlated:

$$\langle \xi_i \rangle = 0, \quad \langle \xi_i(t)\xi_j(t') \rangle = Q\delta_{ij}\delta(t - t'). \qquad (5)$$

The Kronecker symbol δ_{ij} is 1, if $i = j$ and zero otherwise, while the Dirac function $\delta(t)$ is defined by $\int_{-\infty}^{\infty} \delta(t')\,dt' = 1$ and $\delta(t) = 0$ for $t \neq 0$.

3 Simulations of a traffic scenario

In the following, we will apply the VDT to three car-following models, namely the intelligent-driver model (IDM) [7], the optimal-velocity model (OVM) [5],

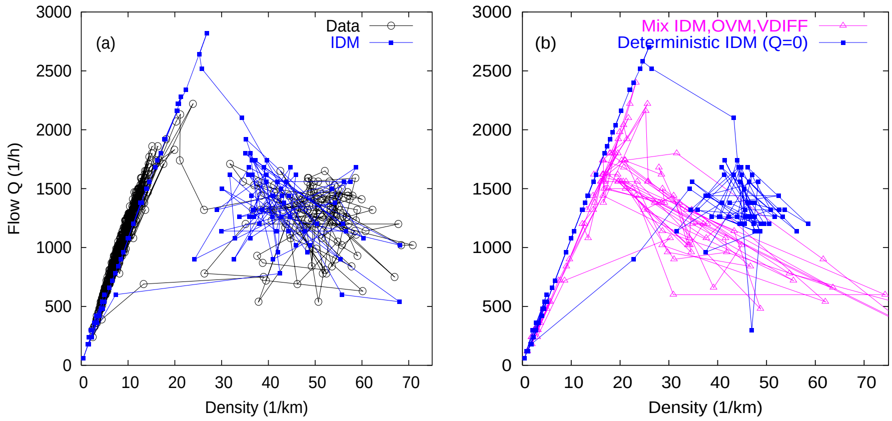

Fig. 3. Empirical and simulated flow-density data obtained from aggregated data (aggregation interval: 60 s) from real and virtual detectors. The empirical curve ("Data") is obtained by aggregating single-vehicle data on the Dutch freeway A9.

and the velocity-difference model (VDIFF) [6], which augments the OVM by a term proportional to the velocity difference. We will also simulate heterogeneous traffic (referred to as MIX) consisting of a mixture of 1/3 IDM, 1/3 OVM, and 1/3 VDIFF. For each model, we assume 80 % 'cars', and 20 % 'trucks' which differ only in the desired velocity (35 m/s and 25 m/s, respectively). For the model equations and parameters, we refer to Ref. [11].

We have simulated a single-lane road section of total length 15 km with an on-ramp of merging length $L_{\rm rmp} = 200$ m located at $x_{\rm rmp} = 12$ km, from which a constant flow of 400 vehicles/h merges to the main road [11]. Instead of explicitely modelling on-ramp lane changes, we have inserted the ramp vehicles centrally into the largest gap within the merging region. At the merging, the velocity was 60% of that of the respective front vehicle.

We have started the simulation with free traffic and increased the traffic demand at the in-flowing boundary linearly from 300 vehicles/h at $t = 0$ s to 3000 vehicles/h at $t = 2400$ s. Afterwards, we decreased the inflow linearly to 300 vehicles/h until $t = 4800$ s. In case the inflow exceeded capacity, we delayed the insertion of new vehicles at the upstream boundary. To enable a direct comparison with detector data, we implemented a 'virtual detector' 4 km upstream of the on-ramp recording the passage time, velocity, and type of each vehicle.

Figure 2 compares the local variation coefficient, Eq. (2) with $n = 5$, as obtained from empirical single-vehicle data, with that obtained from the virtual detector when simulating the VDT with the three mentioned models and with the model mix. Both the decrease with density for low densities and the distinct increase at $\rho \approx 35$ veh./km are reproduced. Notice that this dependence on the density is an emergent result and not contained in the model assumptions.

Figure 3 (a) shows that the fundamental diagram obtained from simulations of the VDT with the IDM agrees, in a statistical sense, quantitatively with empirical observations. Remarkably, wide scattering is even observed in a completely deterministic simulation (Fig. 3 (b)).

Figure 4 shows simulated statistical single-vehicle properties. Both the wide time-gap distribution and the shift of the maximum with traffic density are reproduced, although the agreement is not quantitative (cf. Fig. 1). The wider distributions obtained for the model mix suggest that heterogeneity in the population of driver-vehicle units plays an essential role. Finally, the distribution of times-to-collision agrees nearly quantitatively with the data for the OVM and the model mix, but not for the IDM. In conclusion, the model mix can reproduce, at least semiquantitatively, the fundamental diagram, the variance function, and the distributions of time headways and times-to-collision.

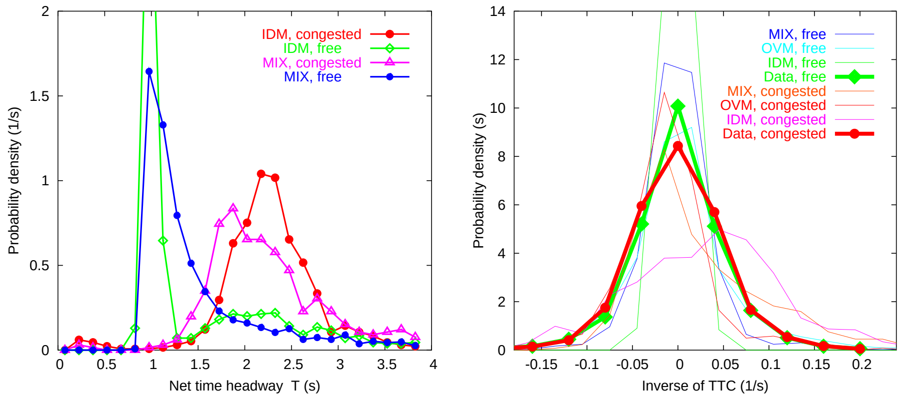

Fig. 4. (a) Distribution of the net time headways of cars following any kind of vehicle (cars or trucks); (b) distribution of the inverse $(v_i - v_{i-1})/s_i$ of the times-to-collision obtained from 'virtual detectors' of VDT simulations with different underlying models.

4 Discussion

In the variance-driven time headway (VDT) model put forward in this paper, the desired safety time headway increases with the local velocity variance. Therefore, when traffic becomes unstable, larger time gaps are held in order to keep up a reasonable level of safety, which is reflected in the nearly unchanged times-to-colision. This causes, on the one hand, a capacity drop. On the other hand, time headways are expecetd to be small in platoons consisting of vehicles driving with similar velocities. Therefore, platoons are rather long-lived and time gaps are much larger between them. This explains (at least, partially) the large scattering of flow-density data [3] in synchronized traffic flow [2].

We note that an understanding of the effects of the velocity variance is crucial for devising measures to avoid traffic breakdowns: The VDT feedback mechanism is triggered most likely near sources of sustained velocity variations, for example in the merging, diverging, or weaving zones near freeway intersections. Particularly, it is essential to avoid merging and diverging maneuvers at high velocity differences, e.g., by increasing the length of the acceleration lane at on-ramps and off-ramps. The simulations illustrate this point: When simulating on-ramp vehicles merging with the velocity of the nearby main-road vehicles (as compared to 60% of this velocity), we have observed a delayed traffic breakdown and sometimes even no breakdown at all for the same traffic demand.

Finally, the distinct increase of the time headways after traffic breakdown allows for vehicle-based options to increase the traffic performance and stability by means of adaptive cruise control systems [12].

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References

1. D. Helbing, “Traffic and related self-driven many-particle systems,” Reviews of Modern Physics **73,** 1067–1141 (2001).
2. B. S. Kerner, *The Physics of Traffic* (Springer, Heidelberg, 2004).
3. K. Nishinari, M. Treiber, and D. Helbing, “Interpreting the wide scattering of synchronized traffic data by time gap statistics,” Phys. Rev. E **68,** 067101 (2003).
4. M. Treiber, A. Hennecke, and D. Helbing, “Derivation, properties, and simulation of a gas-kinetic-based, non-local traffic model,” Phys. Rev. E **59,** 239–253 (1999).
5. M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, “Dynamical model of traffic congestion and numerical simulation,” Phys. Rev. E **51,** 1035–1042 (1995).
6. R. Jiang, Q. Wu, and Z. Zhu, “Full velocity difference model for a car-following theory,” Physical Review E **64,** 017101 (2001).
7. M. Treiber, A. Hennecke, and D. Helbing, “Congested traffic states in empirical observations and microscopic simulations,” Phys. Rev. E **62,** 1805–1824 (2000).
8. P. G. Gipps, “A behavioural car-following model for computer simulation,” Transp. Res. B **15,** 105–111 (1981).
9. M. Treiber, A. Kesting, and D. Helbing, “Delays, inaccuracies and anticipation in microscopic traffic models,” Physica A **359,** 729–746 (2006).
10. D. Helbing and M. Treiber, “Fokker-Planck equation approach to vehicle statistics,” (2003), `cond-mat/0307219`.
11. M. Treiber, A. Kesting, and D. Helbing, “Understanding widely scattered traffic flows, the capacity drop, platoons, and times-to-collision as effects of variance-driven time gaps,” preprint `physics/0508222` (2005).
12. A. Kesting, M. Treiber, M. Schönhof, F. Kranke, and D. Helbing, "’Jam-avoiding’ adaptive cruise control (ACC) and its impact on traffic dynamics”, in this volume.